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A Variable Control Chart Based on Process Capability Index Under Generalized Multiple Dependent State Sampling

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ABSTRACT This paper proposed a process capability index-based control chart under the new extended form of multiple-dependent state sampling (MDS) named generalized MDS (GMDS). The scheme is based on inner and outer control limits and utilizes the previous state of the samples. The performance comparisons of the proposed chart with the existing charts are made by using out-of-control ARL. The simulation study showed the superiority of the proposed chart over the existing PCI-based control charts under Shewhart and MDS schemes. An empirical illustration is also given to demonstrate the application of the proposed chart.

INDEX TERMS Control charts, capability indice, quality control, sampling plans, simulation.

I. INTRODUCTION

Control Chart is an important statistical tool in statistical process control (SPC) used to monitor the process variation. Generally, the process variation can be categorized into the common cause of variation and special or assignable cause of variation. The existence of the common cause of variation does not make the process out-of-control which is considered to be an inherent part of any production process [1]. The typical control charts are based on two horizontal lines named an upper control limit (UCL) and lower control limit (LCL). The process is declared to be out-of-control if the monitoring statistic falls outside these limits which indicate the presence of assignable cause of variation in the production line. SPC has made tremendous progress after the construction of the Shewhart control chart [2]. Afterward, the cumulative sum (CUSUM) control chart by [3] and exponentially moving average EWMA control charts by [4] have made considerable improvement in SPC.

The most enviable state in SPC is that a process is in-control and capable. The data from an in-control process can be used to compute the future performance of the process with the help of process capability indices (PCI). Hence, the stability and the ability of the manufacturing process are evaluated in two stages namely control charts and process capability indices. The PCI has been extensively used measure in the manufacturing industry to determine whether a production process is capable of manufacturing items according to pre-specified quality requirements. Several PCIs are available to estimate the capability of the manufacturing process such as C_a, C_p, C_{pk}, C_{pm} and C_{pmk} [5]. The larger value of PCI corresponds to higher process yield and lesser probable loss. Extensive literature is available on PCI. A detailed review paper by [6] and a bibliography of process capability papers by [7] and Yum and Kim [8] are excellent sources. For further details on PCI see [9]–[23].

The control charts are also used to determine whether the process is capable of producing the quality of products that meet the pre-specified standards. The goal is attained when the control chart is integrated with PCI. In this framework, Subramani and Balamurali [24] proposed a PCI based control chart which combines the two-stage process into a single stage process for online process control. The chart is based on process capability ratios C_p and C_{pk} and used the range statistic as an estimate of process standard deviation.

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Reference [25] designed an efficient control chart based on C_p to monitor and evaluate the process capability for normal and non-normal processes. The scheme used Downton's statistic as an alternative to range statistic for estimating process standard deviation. For more details on PCI based chart, see [26], [27]. More details on the application of the control charts can be seen in [28]–[31].

The control charts mentioned above are designed based on the assumption that samples are taken from simple random sampling. However, in practice, a single sampling scheme is not essentially desirable and use of more structured sampling schemes can produce control charts which are efficient to their counterparts. In literature, more structured sampling schemes are available to improve the performance of the control charts in detecting shifts quickly such as ranked set sampling (RSS) introduced by McIntyre [32], repetitive sampling (RS) by Sherman [33] and multiple dependent state sampling (MDS) by Wortham and Baker [34]. In the context of process capability analysis, Ahmad *et al.* [35] proposed a PCI-based control chart using repetitive sampling scheme. The scheme used the process capability index C_p as a monitoring statistics and sample range as an estimate of standard deviation. Nevertheless, the use of range statistics is not effective for large sample sizes as it is based on only two extreme values of the sample. Reference [36] investigated the use of unbiased estimator $\hat{\sigma} = \bar{s}/c_4$ of standard deviation for computing PCI based control charts under repetitive sampling plan for normal process. The scheme performs better results in terms of ARL and SDRL, especially for smaller shifts. For more details on the implementation of well-structured sampling schemes in the constructions of control charts, the readers may refer to [35] and [37]–[41].

Motivated by the attractive features of a more structured sampling scheme MDS; we have used generalized MDS scheme in designing PCI control chart by adding a new parameter that increases the sensitivity of the chart. The scheme is named as generalizing multiple dependent sampling (GMDS). The scheme is integrated to develop PCI based chart under the normal process. The performance of the proposed chart is evaluated by using out-of-control ARL obtained through a simulation study. An empirical illustration is provided for practical implementation of the proposed chart.

The rest of this paper is organized as follows: Section 2 presents the structures of PCI based control chart under GMDS scheme and an algorithm is also provided in this section. Performance evaluations of proposed schemes are given in section 3. Comparisons with the existing chart using *ARL*s, simulation and illustrative examples are made in section 4 and finally, some conclusions are drawn in section 5.

II. DESIGNING OF THE PROPOSED CONTROL CHART

In this section, a new process capability index based control chart is proposed under the extended form of multiple dependent state (MDS) scheme named as generalized multiple dependent state sampling (GMDS). The proposed GMDS scheme based on PCI control chart consists of inner and outer control limits. In order to derive the probabilities of in-control and out-of-control processes, the following assumptions are used:

(1) For the in-control process, the quality characteristic (X) follows a normal distribution with mean μ and variance σ^2.
(2) It is assumed that there exist an upper specification limit (USL) and lower specifications limit (LSL) such that an item beyond these limits is considered to be defective.

The two pairs of control limits for the proposed control chart are defined as follows:

$$LCL_1 = E(\hat{C}_s) - k_1\sqrt{Var(\hat{C}_s)}$$
$$UCL_1 = E(\hat{C}_s) + k_1\sqrt{Var(\hat{C}_s)}$$
$$LCL_2 = E(\hat{C}_s) - k_2\sqrt{Var(\hat{C}_s)}$$
$$UCL_2 = E(\hat{C}_s) + k_2\sqrt{Var(\hat{C}_s)}$$

where k_1 and k_2 are the control chart coefficients and determine in such a way that a specified in-control ARL is achieved. The following steps are involved in the construction of the proposed control chart:

Step 1: Take a random sample of size n at each subgroup and measure its quality characteristic (X). Calculate the sample mean $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and sample variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$ from each subgroup. Then compute the estimate of the process capability index $\hat{C}_s$ (either $\hat{C}_{pu}$ or $\hat{C}_{pl}$)

$$\hat{C}_{pk} = \min\left\{\frac{USL-\bar{X}}{3S}, \frac{\bar{X}-LSL}{3S}\right\} \tag{1}$$

Step 2: Declare the process as in-control if $LCL_2 \leq \hat{C}_s \leq UCL_2$ and declare the process as out-of-control if, $\hat{C}_s > UCL_1$ or $\hat{C}_s < LCL_1$. Otherwise, go to step 3.

Step 3: Declare the process as in-control if at least k out of m preceding samples statistics $\hat{C}_s$ are plotted inside the inner control limits i.e. $LCL_2 \leq \hat{C}_s \leq UCL_2$.

Suppose a random sample of size n is taken from a stable process and estimate the indices C_{pl} and C_{pu} which are defined as follows:

$$\hat{C}_{pl} = \frac{\bar{x}-LSL}{3s}, \quad \hat{C}_{pu} = \frac{USL-\bar{x}}{3s}$$

where $\bar{x}$ is the sample mean and s is the sample standard deviation. For normally distributed data, Chou and Owen [42] showed that the estimator $\hat{C}_s$ (either $\hat{C}_{pl}$ or $\hat{C}_{pu}$) is distributed as $t_{n-1,\,\delta}/3\sqrt{n}$, where $t_{n-1,\,\delta}$ is a non-central t

distribution with $(n-1)$ degrees of freedom and non-central parameter $\delta = 3\sqrt{n}C_s$.

Hence, the probability density function of $\hat{C}_s$ can be expressed as

$$f_{\hat{C}_s}(x) = \frac{3\sqrt{n/(n-1)}2^{-n/2}}{\sqrt{\pi}\Gamma[(n-1)/2]}\int_0^\infty t^{(n-2)/2} \times \exp\left\{\frac{-1}{2}\left[t+\left(\frac{3x\sqrt{nt}}{\sqrt{n-1}}-\delta\right)^2\right]\right\}dt \quad (2)$$

where $\delta = 3\sqrt{n}C_s$. The cumulative distribution function of $\hat{C}_s$ is given as:

$$F_{\hat{C}_s}(x) = \frac{1}{2^{(n-3)/2}\Gamma[(n-1)/2]}\int_0^\infty t^{(n-2)/2}e^{-t^2/2}\frac{1}{\sqrt{2\pi}} \times \int_0^{3\sqrt{n}tx/\sqrt{(n-1)}} exp\left[-\frac{(\mu-\delta)^2}{2}\right]dudt \quad (3)$$

The Control Limits for $\hat{C}_s$ chart

We know that $E[t_{n-1,\ \delta}] = \delta\sqrt{(n-1)/2}\frac{\Gamma[(n-2)/2]}{\Gamma[(n-1)/2]}$ and $Var(t_{n-1,\ \delta}) = \frac{(n-1)(1+\delta^2)}{n-3} - \frac{\delta^2(n-1)}{2}\left(\frac{\Gamma[(n-2)/2]}{\Gamma[(n-1)/2]}\right)^2$.

Since $\hat{C}_s$ is distributed as $t_{n-1,\ 3\sqrt{n}C_s}/3\sqrt{n}$, we can derive

$$E(\hat{C}_s) = E[t_{n-1,\ 3\sqrt{n}C_s}/3\sqrt{n}] = \frac{1}{3\sqrt{n}}\left(3\sqrt{n}C_s\sqrt{(n-1)/2}\frac{\Gamma[(n-2)/2]}{\Gamma[(n-1)/2]}\right) = C_s\sqrt{(n-1)/2}\frac{\Gamma[(n-2)/2]}{\Gamma[(n-1)/2]} \quad (4)$$

$$Var(\hat{C}_s) = Var[t_{n-1,\ 3\sqrt{n}C_s}/3\sqrt{n}] = \frac{1}{9n}\left\{\frac{(n-1)(1+9nC_s^2)}{(n-3)} - \frac{9nC_s^2(n-1)}{2} \times \left(\frac{\Gamma[(n-2)/2]}{\Gamma[(n-1)/2]}\right)^2\right\}. \quad (5)$$

The outer control limits for $\hat{C}_s$ are defined as equation, shown at the bottom of this page, where

$$a_n = \sqrt{(n-1)/2}\frac{\Gamma[(n-2)/2]}{\Gamma[(n-1)/2]}, \quad d_n = \frac{n-1}{n-3}.$$

Similarly, the inner control limits can be written as:

$$UCL_2 = C_s a_n + k_2\sqrt{d_n/9n + C_s^2(d_n - a_n^2)}$$
$$LCL_2 = C_s a_n - k_2\sqrt{d_n/9n + C_s^2(d_n - a_n^2)}$$

The probability that the proposed control chart considered as in-control is given as follows:

$$P_{in}^0 = P_a + P_s\left\{\sum_{j=k}^{m}\binom{m}{j}P_a^j(1-P_a)^{m-j}\right\} \quad (6)$$

where

$$P_a = p(LCL_2 \le \hat{C}_s \le UCL_2|C_s^0) = p(\hat{C}_s > LCL_2|C_s^0) - p(\hat{C}_s > UCL_2|C_s^0) = p\left(t_{n-1,\delta} \ge 3\sqrt{n}LCL_2|C_s^0\right) - p\left(t_{n-1,\delta} \ge 3\sqrt{n}UCL_2|C_s^0\right).$$

$$UCL_1 = E(\hat{C}_s) + k_1\sqrt{Var(\hat{C}_s)} = C_s\sqrt{(n-1)/2}\frac{\Gamma[(n-2)/2]}{\Gamma[(n-1)/2]} + k_1\sqrt{\frac{1}{9n}\left\{\frac{(n-1)(1+9nC_s^2)}{(n-3)} - \frac{9nC_s^2(n-1)}{2}\left(\frac{\Gamma[(n-2)/2]}{\Gamma[(n-1)/2]}\right)^2\right\}} = C_s a_n + k_1\sqrt{\frac{1}{9n}\{d_n(1+9nC_s^2) - 9nC_s^2a_n^2\}} = C_s a_n + k_1\sqrt{d_n/9n + C_s^2(d_n - a_n^2)}$$

$$LCL_1 = E(\hat{C}_s) - k_1\sqrt{Var(\hat{C}_s)} = C_s\sqrt{(n-1)/2}\frac{\Gamma[(n-2)/2]}{\Gamma[(n-1)/2]} - k_1\sqrt{\frac{1}{9n}\left\{\frac{(n-1)(1+9nC_s^2)}{(n-3)} - \frac{9nC_s^2(n-1)}{2}\left(\frac{\Gamma[(n-2)/2]}{\Gamma[(n-1)/2]}\right)^2\right\}} = C_s a_n - k_1\sqrt{\frac{1}{9n}\{d_n(1+9nC_s^2) - 9nC_s^2a_n^2\}} = C_s a_n - k_1\sqrt{d_n/9n + C_s^2(d_n - a_n^2)}$$

$$P_s = p(LCL_1 \leq \hat{C}_s \leq LCL_2|C_s^0) + p(UCL_2 \leq \hat{C}_s \leq UCL_1|C_s^0)$$
$$= p(\hat{C}_s > LCL_1|C_s^0) - p(\hat{C}_s > LCL_2|C_s^0) + p(\hat{C}_s > UCL_2|C_s^0) - p(\hat{C}_s > UCL_1|C_s^0)$$
$$= p\left(t_{n-1,\delta} \geq 3\sqrt{n}LCL_1|C_s^0\right) - p\left(t_{n-1,\delta} \geq 3\sqrt{n}LCL_2|C_s^0\right) + p\left(t_{n-1,\delta} \geq 3\sqrt{n}UCL_2|C_s^0\right) - p\left(t_{n-1,\delta} \geq 3\sqrt{n}UCL_1|C_s^0\right)$$

Therefore, the in-control average run length (ARL), denoted as ARL_0, of the proposed control chart can be obtained as follows:

$$ARL_0 = \frac{1}{1 - P_{in}^0} \tag{7}$$

The out-of-control ARL performance can be derived in a similar manner as explained previously. Suppose that process mean has shifted from μ_0 to μ_1 where $\mu_1 = \mu_0 + c\sigma$ and c constant is the magnitude of the shift. The probability that the process is declared to be in-control for the shifted process is given as follows:

$$P_{in}^1 = P_{a1} + P_{s1}\left\{\sum_{j=k}^{m}\binom{m}{j}P_{a1}^j(1-P_{a1})^{m-j}\right\} \tag{8}$$

where

$$P_{a1} = p(LCL_2 \leq \hat{C}_s \leq UCL_2|C_s^1) = p(\hat{C}_s > LCL_2|C_s^1) - p(\hat{C}_s > UCL_2|C_s^1)$$
$$= p\left(t_{n-1,\delta} \geq 3\sqrt{n}LCL_2|C_s^1\right) - p\left(t_{n-1,\delta} \geq 3\sqrt{n}UCL_2|C_s^1\right).$$

$$P_{s1} = p(LCL_1 \leq \hat{C}_s \leq LCL_2|C_s^1) + p(UCL_2 \leq \hat{C}_s \leq UCL_1|C_s^1)$$
$$= p(\hat{C}_s > LCL_1|C_s^1) - p(\hat{C}_s > LCL_2|C_s^1) + p(\hat{C}_s > UCL_2|C_s^1) - p(\hat{C}_s > UCL_1|C_s^1)$$
$$= p\left(t_{n-1,\delta} \geq 3\sqrt{n}LCL_1|C_s^1\right) - p\left(t_{n-1,\delta} \geq 3\sqrt{n}LCL_2|C_s^1\right) + p\left(t_{n-1,\delta} \geq 3\sqrt{n}UCL_2|C_s^1\right) - p\left(t_{n-1,\delta} \geq 3\sqrt{n}UCL_1|C_s^1\right)$$

Therefore, ARL_1 is given by

$$ARL_1 = \frac{1}{1 - P_{in}^1}. \tag{9}$$

The summary of the algorithm is as follows:

Step 1: Generate a random sample of size n for each subgroup from a normal distribution with the specified in-control process.

Step 2: Compute $\hat{C}_s$ for each subgroup.

Step 3: Fix the in-control ARL, say r_0.

Step 4: Determine the values of control chart coefficients k_1 and k_2 such that $ARL_0 \geq r_0$.

Step 5: Choose that value of limit coefficients k_1 and k_2 from step-4 for which ARL_0 is minimum i.e. $ARL_0 = r_0$.

Step 6: Obtain the out-of-control ARLs for various shifts by using the values of limit coefficients k_1 and k_2 obtained in Step-5.

III. RESULTS AND DISCUSSION

This section establishes the performance of the proposed control chart using the average run length (ARL). The ARL is the expected number of samples until a control chart first signals. For comparison purpose, we developed both in-control average run length (ARL_0) and out-of-control average run length (ARL_1). The proposed control chart depends on the chart coefficients k_1 and k_2, sample size *n, m* and k. The Monte Carlo simulation is used to determine the values of the limit coefficients k_1 and k_2 for various values of *n*, *k* and *m* at a specific value of ARL_0=370. Furthermore, the out-of-control run lengths (ARL_1) are also obtained for various mean shifts. Results are displayed in Tables 1 and 2. Results demonstrate that the ARL_1 values decrease considerably with the increase in n and m except for $m = k$ since in this case, the GMDS scheme reduces to MDS scheme. Results also indicate that ARL_1 decreases quickly by increasing a small mean shift in the process. It can be noted that in case of $k = m - j$; $j = 1, 2, ..., m - 2$, the proposed chart perform better than the case of $k = m$ (the usual MDS chart), particularly for smaller process shifts.

IV. COMPARATIVE STUDY

This section has comprehended with two subsections. Section 4.1 will affirm the application of the proposed control chart in real life environment and subsequently, in Section 4.2, we will discuss the performance of the proposed control charts using a simulation study. The performance evaluation of the proposed scheme with the existing process capability index C_{pk} (or C_s) based control charts under classical Shewhart and multiple dependent state schemes are constructed with the help of out-of-control average run length. The sensitivity of the proposed chart is

TABLE 1. ARLs of the proposed control chart for $n = 5$.

Shift (c)	k_1=5.5707 k_2=3.8828	k_1=5.7263 k_2=1.7355	k_1=5.6231 k_2=4.3735	k_1=5.3922 k_2=3.2858	k_1=5.3922 k_2=3.2858	k_1=5.8043 k_2=3.9659	k_1=5.4249 k_2=2.9755
	m=3, k=3	m=3, k=2	m=4, k=4	m=4, k=3	m=4, k=2	m=5, k=5	m=5, k=4
	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
0.0	370.01	370.01	370.00	370.01	370.01	370.01	370.00
0.1	278.83	264.41	285.25	276.39	274.19	289.06	275.98
0.2	212.64	186.52	222.52	209.31	206.17	227.84	208.18
0.3	163.97	130.10	175.48	160.61	157.35	181.06	158.56
0.4	127.74	90.04	139.79	124.70	121.84	144.97	121.68
0.5	100.46	62.17	112.38	97.84	95.65	116.89	93.90
0.6	79.71	43.08	91.13	77.45	76.06	94.87	72.72
0.7	63.76	30.14	74.48	61.77	61.21	77.48	56.45
0.8	51.41	21.38	61.33	49.57	47.82	63.66	43.88
0.9	41.75	15.45	50.85	39.98	38.98	52.62	34.17
1.0	34.16	11.39	42.44	32.38	32.03	43.74	26.67
1.5	13.91	3.47	18.80	11.92	11.19	18.92	8.48
2.0	6.72	1.75	9.54	5.06	4.88	9.45	3.57
2.5	3.83	1.25	5.50	2.68	2.10	5.43	2.06
3.0	2.52	1.09	3.56	1.77	1.58	3.54	1.50

TABLE 1. *(Continued.)* ARLs of the proposed control chart for $n = 5$.

Shift (c)	k_1=4.9295 k_2=3.4057	k_1=5.0297 k_2=2.4091	k_1=5.9533 k_2=4.0721	k_1=5.6199 k_2=4.7122	k_1=5.5438 k_2=4.0703	k_1=5.5285 k_2=2.5243	k_1=5.3449 k_2=1.2949
	m=5, k=3	m=5, k=2	m=6, k=6	m=6, k=5	m=6, k=4	m=6, k=3	m=6, k=2
	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
0.0	370.00	370.01	370.02	370.01	370.01	370.01	370.01
0.1	270.83	268.96	303.37	289.95	284.86	283.92	274.65
0.2	196.29	195.99	250.28	229.99	222.29	220.87	206.34
0.3	145.29	144.52	207.67	184.51	175.68	174.03	156.54
0.4	109.06	108.17	173.22	149.60	140.49	138.76	119.18
0.5	82.73	82.19	145.21	122.48	113.60	111.84	90.18
0.6	63.88	63.34	122.29	101.20	92.81	91.02	66.95
0.7	50.16	49.40	103.45	84.32	76.55	74.71	48.25
0.8	39.03	38.89	87.87	70.81	63.71	61.74	33.62
0.9	31.40	30.80	74.95	59.90	53.45	51.28	22.84
1.0	25.58	24.44	64.18	51.01	45.19	42.73	15.37
1.5	8.55	7.34	31.29	24.83	21.40	16.76	2.97
2.0	3.51	2.54	16.75	13.42	11.20	6.21	1.36
2.5	1.88	1.40	9.84	7.85	6.21	2.75	1.07
3.0	1.34	1.11	6.32	4.95	3.68	1.65	1.01

demonstrated with two examples. Comparisons of the average run length of the existing C_{pk} control chart under Shewhart, MDS and GMDS schemes are exhibited in Table 3 for different mean shifts at n=10, m=4 (other comparison tables are with authors). The comparison reveals that the ARL_1 of the proposed chart is significantly smaller than the existing

TABLE 1. *(Continued.)* ARLs of the proposed control chart for $n = 5$.

Shift (c)	k_1 =5.9890 k_2 =4.0464	k_1 =5.9673 k_2 =2.5122	k_1 =5.9822 k_2 =1.6514	k_1 =5.6179 k_2 =2.6265	k_1 =5.5382 k_2 =1.4055	k_1 =5.3091 k_2 =1.3531	k_1 =5.8705 k_2 =4.9975
	m=7, k=7	m=7, k=6	m=7, k=5	m=7, k=4	m=7, k=3	m=7, k=2	m=8, k=8
	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
0.0	370.00	370.01	370.01	370.01	370.01	370.02	370.01
0.1	302.14	297.60	296.82	287.14	283.91	273.78	312.90
0.2	248.21	239.39	236.25	224.16	220.33	205.46	266.20
0.3	205.05	192.40	186.19	175.07	172.37	156.37	227.73
0.4	170.27	154.41	145.17	135.84	135.17	120.47	195.84
0.5	142.08	123.72	112.05	105.94	105.40	93.57	169.23
0.6	119.11	99.01	85.79	81.95	80.90	72.73	146.91
0.7	100.29	79.20	65.36	61.16	60.55	55.97	128.08
0.8	84.81	63.38	49.72	44.11	43.97	42.09	112.12
0.9	72.02	50.81	37.90	31.22	31.09	30.64	98.51
1.0	61.41	40.86	29.05	21.72	21.62	21.58	86.87
1.5	29.43	14.96	8.98	4.71	4.21	3.77	48.59
2.0	15.62	6.70	3.87	1.98	1.69	1.47	29.10
2.5	9.18	3.73	2.26	1.34	1.19	1.08	18.47
3.0	5.94	2.49	1.64	1.14	1.06	1.02	12.37

TABLE 1. *(Continued.)* ARLs of the proposed control chart for $n = 5$.

Shift (c)	k_1 =5.8348 k_2 =3.5917	k_1 =5.8233 k_2 =2.4675	k_1 =5.6766 k_2 =2.7611	k_1 =5.6636 k_2 =1.9664	k_1 =5.3454 k_2 =2.3033	k_1 =5.3409 k_2 =1.0944
	m=8, k=7	m=8, k=6	m=8, k=5	m=8, k=4	m=8, k=3	m=8, k=2
	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
0.0	370.01	370.00	370.01	370.01	370.01	370.02
0.1	310.83	309.28	294.61	293.46	275.15	274.90
0.2	262.68	259.91	237.16	235.37	207.63	207.10
0.3	223.17	219.35	192.86	190.71	159.02	158.10
0.4	190.50	185.67	158.30	155.95	123.54	121.91
0.5	163.30	157.45	131.05	128.54	97.27	94.30
0.6	140.48	133.57	109.33	106.59	77.55	72.23
0.7	121.23	113.23	91.81	88.74	62.54	53.80
0.8	104.89	95.79	77.53	73.95	50.96	38.30
0.9	90.96	80.81	65.74	61.47	41.89	25.94
1.0	79.02	67.94	55.90	50.75	34.65	16.98
1.5	39.85	27.29	24.50	16.35	13.08	2.73
2.0	20.71	11.20	9.91	5.02	4.19	1.25
2.5	11.35	5.36	4.33	2.23	1.81	1.04
3.0	6.76	3.12	2.39	1.45	1.23	1.01

schemes for a choice of shifts considered, various values of m and various sample sizes. The ARL curves of the C_{pk} chart under Shewhart, MDS and GMDS schemes are shown in Figure 1 for various mean shifts at $n = 10$, $m = 6$. There is

TABLE 2. ARLs of the proposed control chart for $n = 10$.

Shift (c)	k_1=4.4499 k_2=3.4838	k_1=4.1956 k_2=3.4306	k_1=4.6009 k_2=2.8239	k_1=3.9897 k_2=2.6442	k_1=4.1077 k_2=1.5403	k_1=4.4662 k_2=3.5657	k_1=4.2589 k_2=3.3191
	m=3, k=3	m=3, k=2	m=4, k=4	m=4, k=3	m=4, k=2	m=5, k=5	m=5, k=4
	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
0.0	370.00	370.02	370.02	370.04	370.03	370.02	370.00
0.1	250.62	225.81	214.88	207.85	206.65	250.12	231.97
0.2	172.60	142.27	126.71	119.77	107.53	171.69	149.52
0.3	120.70	92.36	76.20	70.44	52.63	119.54	98.82
0.4	85.62	61.66	46.95	41.94	25.01	84.35	66.75
0.5	61.55	42.24	29.76	25.19	12.24	60.30	45.94
0.6	44.84	29.62	19.48	15.32	6.50	43.66	32.10
0.7	33.09	21.22	13.20	9.55	3.84	32.04	22.72
0.8	24.74	15.49	9.28	6.19	2.53	23.84	16.26
0.9	18.75	11.51	6.76	4.22	1.86	18.00	11.78
1.0	14.41	8.69	5.11	3.05	1.49	13.80	8.66
1.5	4.82	2.76	2.00	1.29	1.03	4.69	2.61
2.0	2.34	1.44	1.31	1.05	1.00	2.37	1.45
2.5	1.54	1.11	1.09	1.00	1.00	1.58	1.14
3.0	1.22	1.03	1.02	1.00	1.00	1.25	1.04

TABLE 2. *(Continued.)* ARLs of the proposed control chart for $n = 10$.

Shift (c)	k_1=4.1701 k_2=3.3928	k_1=4.0126 k_2=3.9401	k_1=4.6182 k_2=3.4353	k_1=4.4037 k_2=3.5655	k_1=4.3386 k_2=3.9648	k_1=4.2633 k_2=3.5045	k_1=4.0586 k_2=2.1799
	m=5, k=3	m=5, k=2	m=6, k=6	m=6, k=5	m=6, k=4	m=6, k=3	m=6, k=2
	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
0.0	370.01	370.03	370.01	370.02	370.01	370.02	370.02
0.1	223.50	211.46	269.42	252.29	242.40	232.93	214.53
0.2	139.56	125.40	197.83	175.51	162.80	150.94	129.01
0.3	89.94	77.21	146.41	124.37	111.94	100.53	80.41
0.4	59.72	49.28	109.20	89.63	78.70	68.72	51.80
0.5	40.77	32.56	82.08	65.57	56.50	48.14	34.32
0.6	28.56	22.22	62.20	48.59	41.38	34.51	23.18
0.7	20.47	15.64	47.54	36.43	30.88	25.29	15.75
0.8	14.97	11.34	36.66	27.57	23.45	18.92	10.62
0.9	11.14	8.45	28.55	21.05	18.11	14.42	7.08
1.0	8.41	6.47	22.46	16.20	14.20	11.18	4.74
1.5	2.57	2.39	8.03	5.10	5.09	3.76	1.30
2.0	1.42	1.39	3.85	2.34	2.40	1.72	1.03
2.5	1.11	1.10	2.36	1.53	1.51	1.19	1.00
3.0	1.02	1.02	1.71	1.22	1.20	1.05	1.00

a significant decrease in the values of ARL curves of the proposed chart in comparison with the other existing charts examined in this work.

A. INDUSTRIAL APPLICATION OF THE PROPOSED CHART

To demonstrate the practical implementation of our proposed control chart, an industrial data taken from [9] is used.

TABLE 2. *(Continued.)* ARLs of the proposed control chart for $n = 10$.

Shift (c)	k_1=4.5458 k_2=3.6829	k_1=4.4658 k_2=3.7341	k_1=4.4275 k_2=2.6940	k_1=4.3392 k_2=1.8857	k_1=4.2776 k_2=1.7043	k_1=4.1046 k_2=1.2518	k_1=4.6465 k_2=3.4814
	m=7, k=7	m=7, k=6	m=7, k=5	m=7, k=4	m=7, k=3	m=7, k=2	m=8, k=8
	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
0.0	370.01	370.01	370.03	370.03	370.00	370.00	370.02
0.1	266.96	265.14	256.53	240.69	234.24	217.32	269.77
0.2	194.29	193.02	180.95	158.86	152.06	130.67	198.18
0.3	143.93	142.61	129.50	104.90	100.38	78.93	146.67
0.4	102.23	106.80	93.67	67.96	66.28	45.84	109.35
0.5	81.75	80.99	68.17	42.88	42.70	24.60	82.16
0.6	63.48	62.11	49.66	26.64	26.31	12.49	62.23
0.7	49.35	48.11	36.04	15.77	15.59	6.50	47.56
0.8	38.92	37.60	26.01	9.26	9.19	3.70	36.69
0.9	29.98	29.62	18.69	5.87	5.61	2.38	28.59
1.0	24.35	23.49	13.44	3.96	3.65	1.73	22.53
1.5	8.65	8.17	3.34	1.39	1.24	1.03	8.19
2.0	3.70	3.59	1.64	1.08	1.03	1.00	4.02
2.5	2.28	2.11	1.23	1.02	1.00	1.00	2.49
3.0	1.66	1.53	1.08	1.00	1.00	1.00	1.80

TABLE 2. *(Continued.)* ARLs of the proposed control chart for $n = 10$.

Shift (c)	k_1=4.4108 k_2=4.1536	k_1=4.4001 k_2=3.5180	k_1=4.3455 k_2=3.5271	k_1=4.2469 k_2=2.7024	k_1=4.0562 k_2=2.7685	k_1=3.9846 k_2=2.1465
	m=8, k=7	m=8, k=6	m=8, k=5	m=8, k=4	m=8, k=3	m=8, k=2
	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
0.0	370.02	370.07	370.01	370.00	370.04	370.02
0.1	253.92	252.04	243.38	231.11	214.36	209.78
0.2	177.81	175.29	164.05	148.72	128.83	123.38
0.3	126.91	124.34	113.17	98.43	80.27	75.40
0.4	92.21	89.83	79.79	66.90	51.76	47.82
0.5	68.14	66.02	57.43	46.59	34.49	31.40
0.6	51.16	49.28	42.15	33.15	23.70	21.27
0.7	38.97	37.28	31.50	23.98	16.75	14.74
0.8	30.10	28.53	23.94	17.50	12.11	10.29
0.9	23.54	22.03	18.47	12.76	8.90	7.12
1.0	18.64	17.13	14.42	9.22	6.58	4.84
1.5	6.79	5.31	4.70	2.11	1.70	1.25
2.0	3.22	2.29	2.01	1.20	1.09	1.01
2.5	1.97	1.49	1.33	1.04	1.01	1.00
3.0	1.45	1.20	1.12	1.01	1.00	1.00

Liquid crystals have been used in various configurations for display applications. Most of the current displays involve the use of either twisted nematic or a super-twisted nematic (STN) liquid crystals display (LCD). In order to control the quality of the STN-LCD production line, a statistical test is conducted based on computing the capability

TABLE 3. Comparison of the ARLs of PCI based control chart under Shewhart, MDS and GMDS schemes for $n = 10$.

Shift (c)	Shewhart Scheme L=3	k_1=4.6182 k_2=3.4353 (MDS) m=6, k=6	k_1=4.4037 k_2=3.5655 m=6, k=5	k_1=4.3386 k_2=3.9648 m=6, k=4
	ARL_1	ARL_1	ARL_1	ARL_1
0.0	370.40	370.01	370.02	370.01
0.1	352.93	269.42	252.29	242.40
0.2	308.43	197.83	175.51	162.80
0.3	253.14	146.41	124.37	111.94
0.4	200.08	109.20	89.63	78.70
0.5	155.22	82.08	65.57	56.50
0.6	119.67	62.20	48.59	41.38
0.7	92.32	47.54	36.43	30.88
0.8	71.55	36.66	27.57	23.45
0.9	55.83	28.55	21.05	18.11
1.0	43.89	22.46	16.20	14.20
1.5	14.97	8.03	5.10	5.09
2.0	6.30	3.85	2.34	2.40
2.5	3.24	2.36	1.53	1.51
3.0	2.00	1.71	1.22	1.20

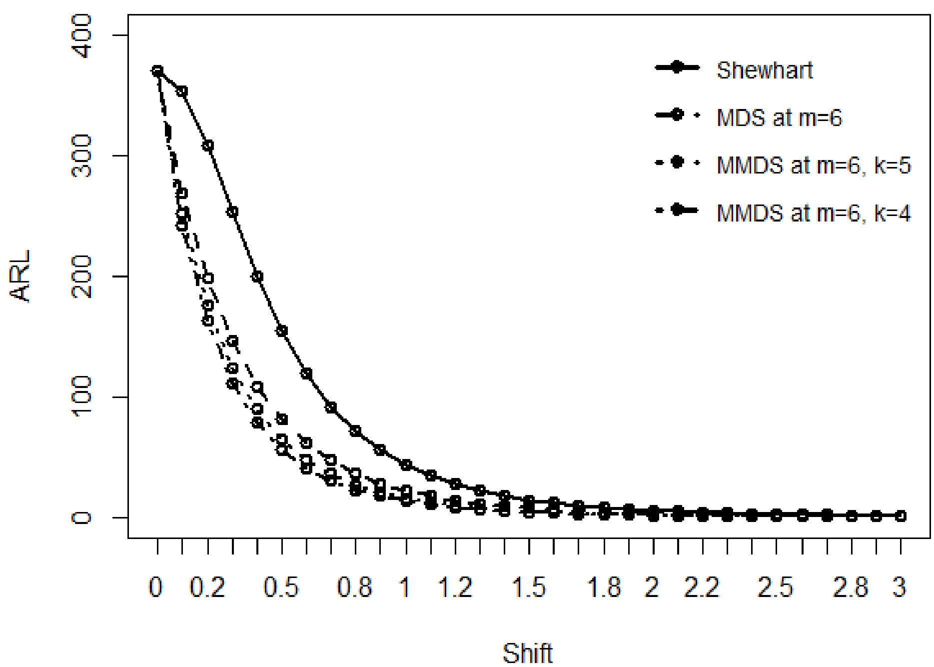

FIGURE 1. PCI based control chart under Shewhart, MDS and GMDS schemes at $n=10$ at $ARL_0=370$.

index C_p. In this regard, fifteen samples each of size 10 related to glass substrate thickness of the LCD was collected in the Science-Based Industrial Park, Taiwan [43]. The upper specification limit, USL, of a glass substrate's thickness is 0.77 mm, the lower specification limit, LSL, of a glass substrate's thickness is 0.63 mm. Reference [9] used this data for the capability of a fuzzy process whereas Pearn and Wu [43] used for Bayesian procedure of capability

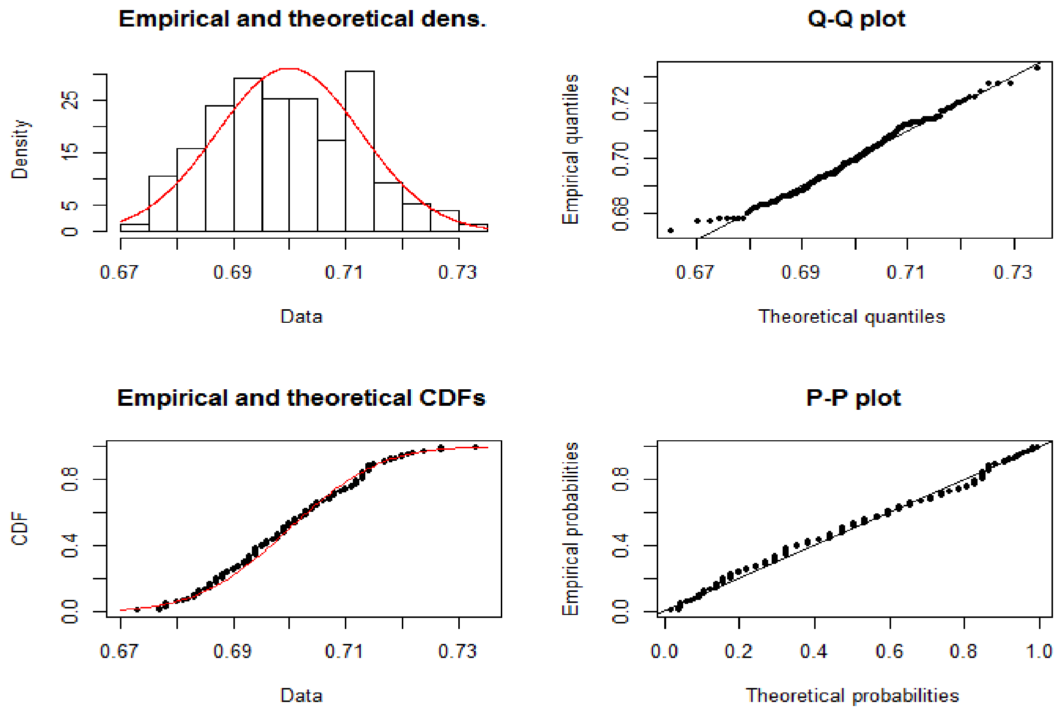

FIGURE 2. Fitting of normal distribution for glass substrate's thickness data.

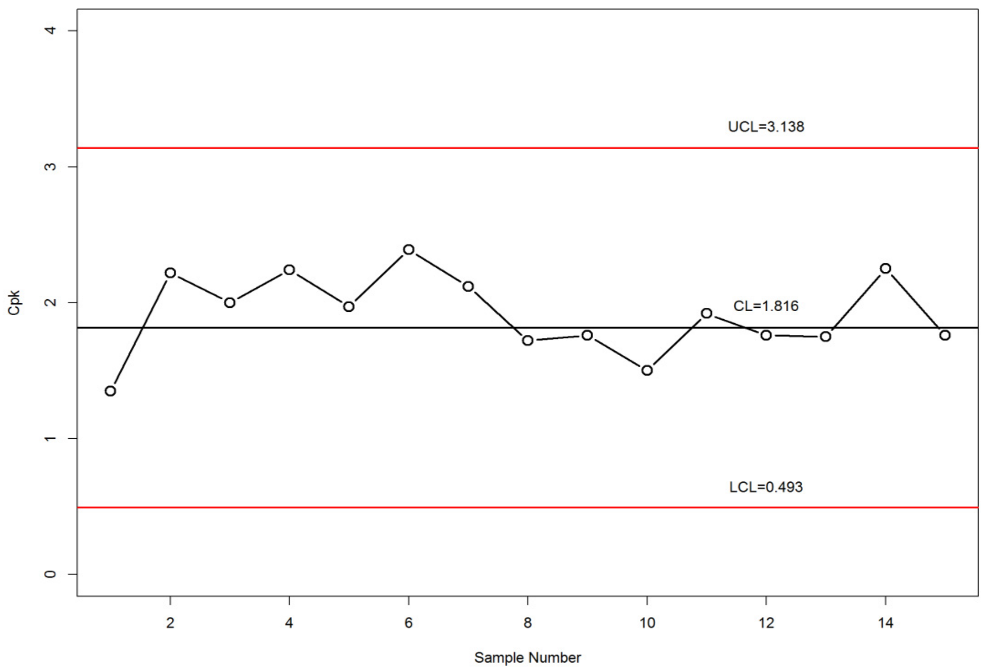

FIGURE 3. Shewhart C_{pk} control chart for glass substrate's thickness data using k=3.

testing. In this article, the same data set is used for our proposed control chart. Before analyzing further, we checked the validity of the model. We plot the empirical and theoretical density, empirical and theoretical CDFs, Q-Q plot and P-P plot for data set showed in Figure 2. We used the Kolmogorov-Smirnov (K-S) tests for data set to the fitted models. It is observed that the K-S distances are 0.0632 with the corresponding p-value is 0.5876. Based on these plots

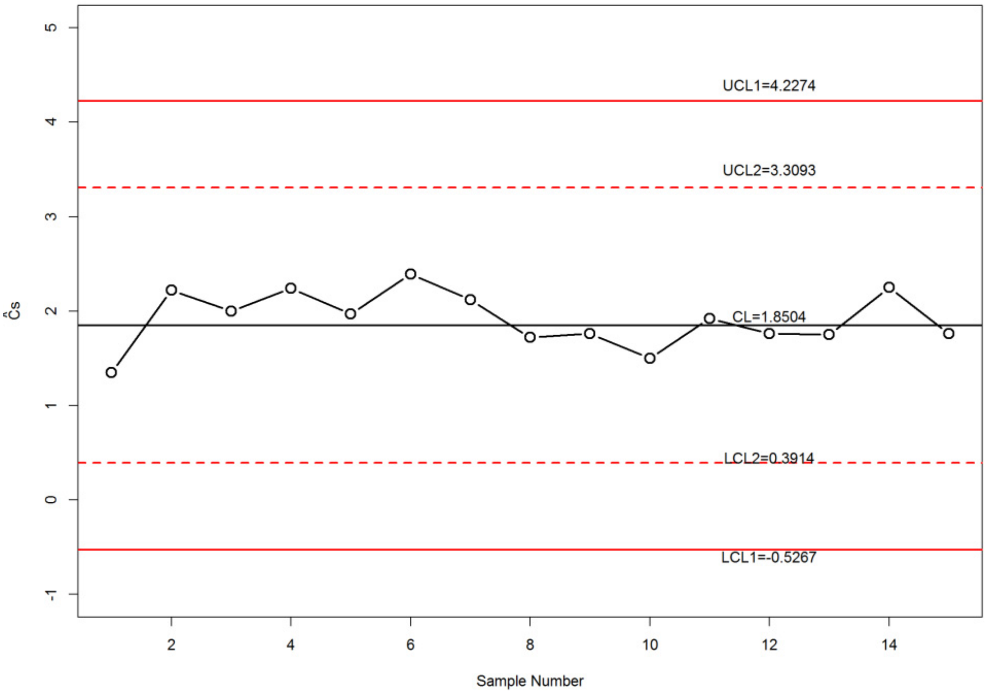

FIGURE 4. PCI based control chart under MDS sampling for glass substrate's thickness data using *m=4*, $k_1 = 4.6009$, $k_2 = 2.8239$ at $ARL_0 = 370$.

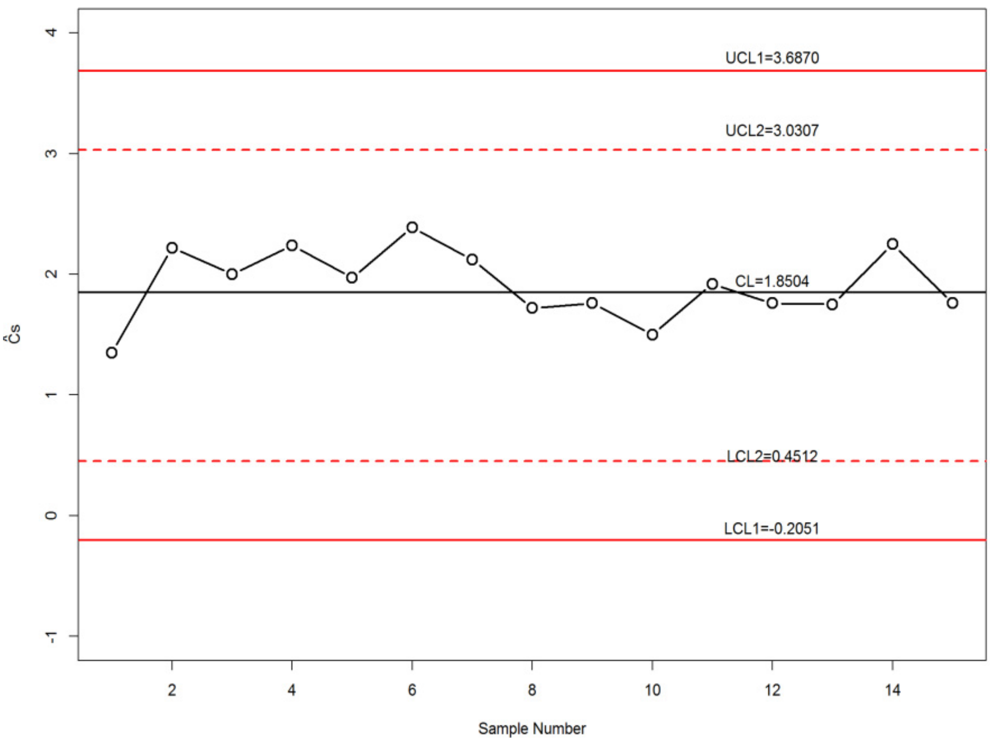

FIGURE 5. Proposed PCI based control chart under GMDS scheme for glass substrate's thickness data using *m=4*, $k_1 = 3.9897$, $k_2 = 2.6442$ at $ARL_0 = 370$.

and K-S test we can conclude that the normal distribution provides a good fit for the given data set. The estimated process capability index C_{pk} of given data is 1.8589.

We construct C_{pk} (or C_s) control charts under Shewhart, MDS and GMDS schemes to evaluate the performance of the charts. Choosing $ARL_0=370$, the Shewhart C_{pk} control

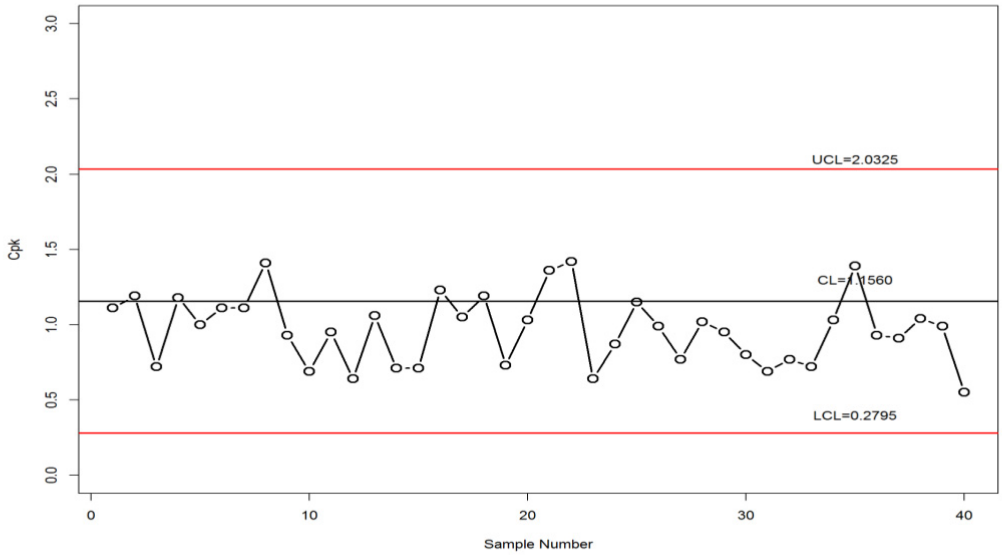

FIGURE 6. The Shewhart C_{pk} chart for simulated data set.

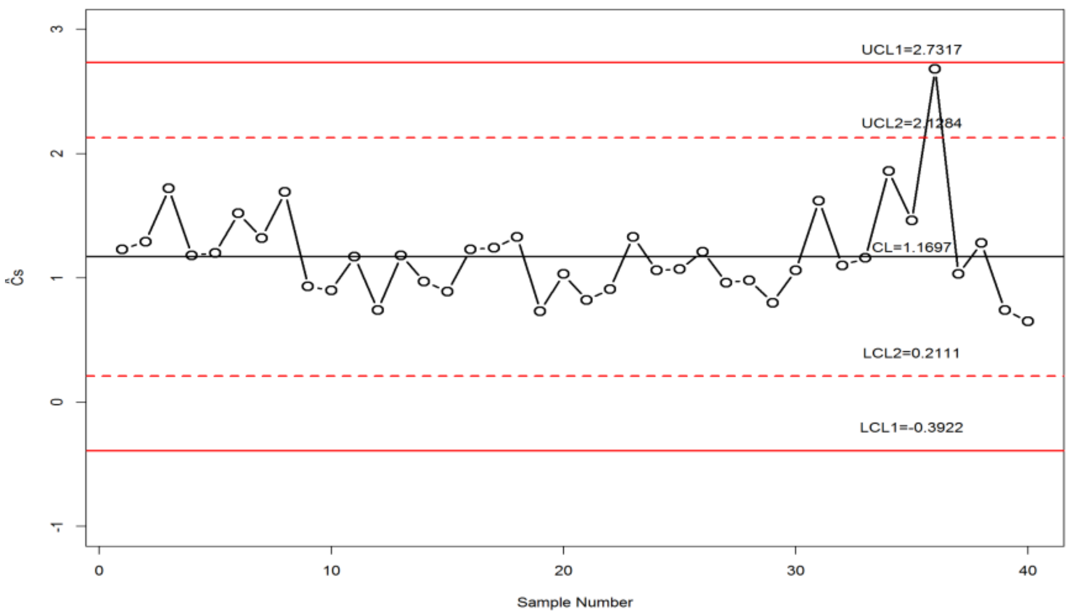

FIGURE 7. PCI based control chart under MDS for simulated data set using $m=4$, $k_1 = 4.6009$, $k_2 = 2.8239$ at ARL_0=370.

chart shown in Figure 3, chart under multiple dependent state scheme at $m=4$, $k_1 = 4.6009$, $k_2 = 2.8239$ shown in Figure 4. Both the charts produce similar results that the process is in-control. The proposed chart is also constructed by declaring that the process will be in-control if at least $k=3$ out of $m=4$, preceding samples fall in the inner control limits. The sample $\hat{C}_s$ are plotted in Figure 5 with the inner and outer control limits of the proposed chart using the limit coefficients $k_1 = 3.9897$, $k_2 = 2.6442$. It is can be observed from Figure 5 that the proposed chart also indicates the in-control state of the production process.

B. SIMULATION STUDY OF PROPOSED CHART

This subsection establishes the performance comparison of the proposed control chart with the existing control charts using simulated data. A data set of 20 subgroups of size 10 each has been generated from the normal distribution for an in-control process with mean 0 and variance 1 and another data set of 20 subgroups of size 10 each has been generated from shifted normal distribution with mean value 0.4 and variance 1. We construct C_{pk} (or C_s) control charts under Shewhart, MDS and GMDS schemes by using the generated data of 40 subgroups for specific ARL0=370.

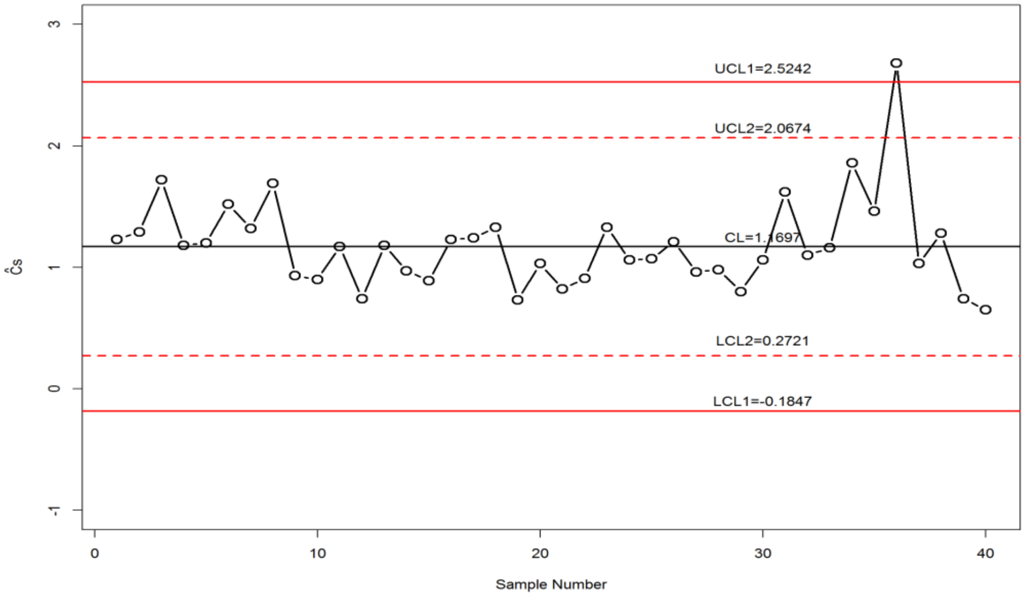

FIGURE 8. **Proposed PCI based control chart under GMDS scheme for simulated data set using m=4, k_1 = 3.9897, k_2 = 2.6442 at ARL_0 = 370.**

Shewhart C_{pk} control chart is constructed with *k=3,* depicted in Figure 6, multiple dependent state C_s chart for *m=4,* $k_1 = 4.6009$, $k_2 = 2.8239$ and displayed in Figure 7. The proposed C_s based control chart is also computed and shown in Figure 8 with parameters *m=4,* $k_1 = 3.9897$, $k_2 = 2.6442$ to guarantee that ARL0=*370*. The classic Shewhart C_{pk} control chart and MDS based C_s chart fail to detect the shift even though Figures 7 shows the upward trend of C_{pk} after subgroup size 30. Whereas, from Figure 8, it is evident that out-of-control signals are detected at samples 36 by the proposed chart. The existing charts fail to detect the shifts whereas the proposed chat requires only 16 samples to detect the shift. This example clearly indicates the efficiency of the proposed chart over the other two existing charts.

V. CONCLUSION

In this article, we have used the generalized multiple dependent scheme to develop process capability indexed based control chart. The proposed scheme is based on two pair of control limits that makes the chart more sensitive in identifying process shift quickly. Shewhart and MDS schemes are special cases of GMDS. The proposed scheme comprises of an estimate of PCI ($\hat{C}_s$) and uses it as a monitoring statistic. The supremacy of the proposed chart is confirmed by comparing ARL curves of the proposed and existing control charts for different shifts. The simulation study also confirmed that the proposed chart identified the shift quickly whereas other existing control charts considered in this study fail to detect this shift. For practical implementation, the proposed procedure is applied to glass substrate thickness data that were collected from the Science-Based Industrial Park, Taiwan. The suggested chart is very useful for manufacturing industry to assess and provide the chances of continuous process improvement. This idea can be used to design control charts based on other measures of PCI.

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Authors' photographs and biographies not available at the time of publication.