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A Time Truncated Moving Average Chart for the Weibull Distribution

SAEED AHMAD DOBBAH ALGHAMDI[1], MUHAMMAD ASLAM[1], KHUSHNOOR KHAN[1], AND CHI-HYUCK JUN[2], (Member, IEEE)
[1]Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah 21551, Saudi Arabia
[2]Department of Industrial and Management Engineering, POSTECH, Pohang 37673, South Korea

Corresponding author: Muhammad Aslam (aslam_ravian@hotmail.com)

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ABSTRACT A control chart of monitoring the number of failures is proposed with a moving average scheme, when the life of an item follows a Weibull distribution. A specified number of items are put on a time truncated life test and the number of failures is observed. The proposed control chart has been evaluated by the average run lengths (ARLs) under different parameter settings. The control constant and the test time multiplier are to be determined by considering the in-control ARL. It is observed that the proposed control chart is more efficient in detecting a shift in the process as compared with the existing time truncated control chart.

INDEX TERMS Control chart, average run length, weibull distribution, defective items.

I. INTRODUCTION

The quality of the product is the key element for maintaining its position in the competitive world markets of manufacturing industry. Walter Shewhart introduced the idea of a control chart during 1920's to monitor the quality of the product, which is still popular and being used with some modification. However, as Stoumbos, *et al.* [1] stated, such simple charts may be inappropriate for detecting a small shift in the process. Thus, during the last few decades, the control chart literature has gained a great attention for the quality control researchers.

In the classical literature, two types of charts have been used in monitoring the production process. When the quality characteristic of interest is measurable, we use variable control charts such as the X-bar chart and S-chart or the R-chart, while attribute (count data) charts such as the p-chart, np-chart are used when the quality characteristic is classified as good or bad. The attribute control charts are particularly important in non-manufacturing quality improvement efforts in which the targeted quality characteristics are impossible to measure on a numerical scale (Montgomery [2]). The number of defects in an item produced by a manufacturing process is common to monitor for improving the quality of the product.

The moving average (MA) chart is quite simple to interpret and to apply because it is based on familiar simple averages of the different sizes (Wong *et al.* [3]). The moving average statistics of the size w is simply the average of w most recent observations. The MA charts have been widely used in the industry for the monitoring of process because they use information obtained from entire sequence of points while Shewhart chart only use current information. Further the MA charts are more sensitive to detect small shift in the process as compared to Shewhart control chart [4]. Due to advantages of MA charts, several authors focus on the designing of MA charts for various situations. [5] designed MA chart for signaling varying location shifts. [3] designed MA-Shewhart control chart. [6] proposed MA chart for joint monitoring of mean and variance in the process. [7] worked on double MA chart. [8] worked on adoptive MA chart. More details about various control charts can be seen in form example, [9] designed X-bar chart using process capability index. [10] control chart for multivariate Poisson distribution using repetitive sampling and [11] designed control chart for Com-Poisson distribution using multiple dependent state sampling.

The performance comparison of the control chart is evaluated by the calculation of the average run length (ARL) which is the average number of samples collected before an out-of-control signal is shown. Therefore, the ARL value is of high interest in the development of any control chart scheme (Knoth [12]). The ARL comparison for in-control and out-of-control processes have been studied by many authors: [13] presented a study on ARL and the average time to signal. [14] studied ARL for a EWMA chart. [15] worked on ARL for multivariate EWMA charts and [16] worked on ARL using a Markov chain. Usually, MA control charts are designed by assuming that the quality of interest follows the normal distribution. But, in practice, it may be possible that data follow some non-normal distribution such as the Weibull distribution [17]. Due to increase in the reliability of the product such as in electronic components, it is not possible to allow a test time long enough. In this situation, the use of time truncated life test control charts may be useful tools to monitor the process. By exploring the literature and best of authors' knowledge, there is no work on design of an MA control chart under the time truncated test for the Weibull distribution. An attribute control chart using a moving average statistic under a time-truncated test is proposed here with the motivation that no chart has been developed for this purpose in the control chart literature to the best of the author's knowledge. The proposed moving average chart employs different sample sizes for the in-control process to monitor the changes in the scale parameter of the Weibull distribution. The result indicates that the proposed control scheme performs well in detecting the small and medium variations in the scale parameter of the Weibull distribution based on different sample sizes of the production process.

The rest of the paper is organized as follows: Section II explains the procedure of the moving average control chart. In Section III the performance of the proposed chart with respect to the ARL values of different shift levels have been discussed. The simulation study is given in Section IV. A real example is given in Section V. In the end, the concluding remarks and the future suggestions have been given.

II. THE PROCEDURE OF MOVING AVERAGE CONTROL CHART

In this section, we will present the design of the proposed control chart for the Weibull distribution. Suppose that the quality of interest follows the Weibull distribution with shape parameter β and scale parameter λ having the cumulative distribution function (cdf) given below.

$$F(t;\lambda,\beta) = 1 - \exp\left(-\left(t/\lambda\right)^{\beta}\right), t \geq 0 \tag{1}$$

As mentioned in [18], [19], [20], and [21] the shape parameter could be assumed to be known in practice. The shape parameter can be estimated from the data if it is unknown. The average life time of the product for the Weibull distribution is given as follows

$$\mu = \left(\lambda/\beta\right)\Gamma\left(1/\beta\right) \tag{2}$$

where Γ (.) is the gamma function. As in [18] the probability that an item fails before experiment time t_0 is given by

$$p_0 = 1 - \exp\left(-\left(t_0/\lambda\right)^{\beta}\right) \tag{3}$$

If we express t_0 as a multiple of the specified mean life μ_0 such that $t_0 = a\mu_0$ for a constant a, then the failure probability of (3) using (2) when the process is in-control when $\mu = \mu_0$ can be rewritten as by

$$p_0 = 1 - \exp\left(-\left(a/\left(\beta/\Gamma\left(1/\beta\right)\right)\right)^{\beta}\right) \tag{4}$$

or

$$p_0 = 1 - \exp\left(-a^{\beta}\left(\Gamma\left(1/\beta\right)/\beta\right)^{\beta}\right) \tag{5}$$

The moving average statistic of size w at time i for the number of failures ($D_i's$) is computed by

$$MA_i = \frac{D_i + D_{i-1} + \ldots D_{i-w+1}}{w} \tag{6}$$

It is important to note that D_i is distributed as a binomial distribution with mean $E(D_i) = np_0$ and variance $\mathrm{Var}(D_i) = np_0(1-p_0)$. The mean and the variance of the statistic MA_i are given as follows

$$E(MA_i) = E(D_i) = np_0 \tag{7}$$

$$\mathrm{Var}(MA_i) = \frac{1}{w^2}\sum_{j=i-w+1}^{i} \mathrm{Var}(D_i) = \frac{1}{w^2}\sum_{j=i-w+1}^{i} np_0(1-p_0) = \frac{np_0(1-p_0)}{w} \tag{8}$$

The proposed control chart is stated as follows:

Step 1: Take a sample of size n from the production process at the i-th subgroup and put them on the test. Count the number of failures (D_i, say) by the specified time$t_0 = a\mu_0$, where μ_0 is the target mean when the process is in-control, and a is a constant. Compute the statistics MA_i using Eq.(6).

Step 2: Declare the process as out-of-control if $MA_i > UCL$ or $MA_i < LCL$. Declare the process as in-control if $LCL \leq MA_i \leq UCL$.

Since D_i is distributed as binomial with n and p_0 when the process is in control, the moving average statistic approximately follows a normal distribution for large i. The proposed control chart reduces to [22] when $w = 1$.The control limits are defined as

$$LCL = np_0 - k\sqrt{\frac{np_0(1-p_0)}{w}} \tag{9}$$

$$UCL = np_0 + k\sqrt{\frac{np_0(1-p_0)}{w}} \tag{10}$$

Here, the control constant k should be determined by considering the target in-control ARL. It is noted that the control limits may be flexibly constructed according to the target in-control ARL and the specified test time. The test time constant a is involved in the failure probability of p_0, so it should be determined at the same time.

III. PERFORMANCE OF PROPOSED CHART USING SIMULATION

This section of the paper describes the ARL values of the proposed control chart. Monte Carlo simulation is the most popular scheme for the evaluation of a control chart in the quality control literature which is used when the theoretical approach is difficult to implement. For this purpose, a large number of statistical sampling experiments are generated (Schaffer and Kim [23]). This simulation approach is used when the distributional properties of the run length characteristics are impossible to handle in a closed form (Fu et al. [24]). The application of the Monte Carlo simulation

TABLE 1. The ARLs of proposed chart with w= 3 for $ARL_0 = 370$, and $\beta = 1.5$.

a	0.246	0.28655	0.25886	0.2445
k	3.0527	3	3	2.99973
	20	30	40	50
δ	ARLs			
1	370.31	369.01	369.30	373.85
0.99	321.95	301.03	308.82	308.84
0.95	159.11	136.63	128.62	110.19
0.92	102.70	78.18	68.07	56.75
0.9	74.42	55.88	44.63	38.89
0.85	37.15	26.40	20.05	16.04
0.8	20.12	13.95	10.24	8.36
0.7	7.71	5.52	4.43	3.77
0.6	4.33	3.53	3.20	3.06
0.5	3.27	3.06	3.01	3.00
0.4	3.03	3.00	3.00	3.00
0.3	3.00	3.00	3.00	3.00
0.2	3.00	3.00	3.00	3.00
0.1	3.00	3.00	3.00	3.00
0.01	3.00	3.00	3.00	3.00

TABLE 2. The ARLs of proposed chart with w= 5 for $ARL_0 = 370$, and $\beta = 1.5$.

a	0.2013	0.264	0.2472	0.2387
k	3	3	3	3
	20	30	40	50
δ	ARLs			
1	370.53	379.51	371.42	369.14
0.99	309.83	301.74	287.73	266.37
0.95	141.96	120.64	101.40	88.15
0.92	82.68	65.61	52.31	43.71
0.9	61.00	45.92	36.03	29.02
0.85	29.13	21.02	16.22	12.89
0.8	16.41	11.77	9.08	7.65
0.7	7.44	6.02	5.40	5.16
0.6	5.34	5.07	5.01	5.00
0.5	5.02	5.00	5.00	5.00
0.4	5.00	5.00	5.00	5.00
0.3	5.00	5.00	5.00	5.00
0.2	5.00	5.00	5.00	5.00
0.1	5.00	5.00	5.00	5.00
0.01	5.00	5.00	5.00	5.00

for evaluation of the control charts have been studied by many authors including ...Sullivan and Woodall [25] and [26]– [28]. The ARLs are estimated by running the proposed scheme using the R-language program. The codes for the R-language program can be requested by the corresponding author. The above-mentioned scheme of the chart has been studied for

TABLE 3. The ARLs of proposed chart with w= 3 for $ARL_0 = 370$, and $\beta = 2$.

	n = 20	n = 30	n = 40	n = 50
a	2.883044	0.286324	2.911037	3.477098
k	0.285821	2.848737	0.290876	0.296667
δ	ARLs			
1.00	370.15	371.15	370.87	370.05
0.99	319.12	306.43	296.39	299.57
0.95	161.56	138.06	121.87	110.98
0.92	101.18	78.48	67.46	57.37
0.90	73.92	54.51	45.89	37.86
0.85	36.82	25.56	19.98	15.99
0.80	19.55	13.36	10.20	8.21
0.70	7.49	5.31	4.32	3.77
0.60	4.13	3.40	3.14	3.06
0.50	3.21	3.03	3.01	3.00
0.40	3.02	3.00	3.00	3.00
0.30	3.00	3.00	3.00	3.00
0.20	3.00	3.00	3.00	3.00
0.10	3.00	3.00	3.00	3.00
0.01	3.00	3.00	3.00	3.00

TABLE 4. The ARLs of proposed chart with w= 5 for $ARL_0 = 370$, and $\beta = 2$.

a	0.3655	0.352	0.3437	0.2773
k	3.00527	3	3.0001	2.99973
	20	30	40	50
δ	ARLs			
1	364.29	382.26	366.22	371.75
0.99	303.53	299.82	307.86	276.49
0.95	139.41	119.79	121.12	92.74
0.92	81.28	65.35	61.94	45.25
0.9	60.09	45.51	41.20	29.92
0.85	28.80	20.65	17.71	13.20
0.8	16.25	11.51	9.67	7.77
0.7	7.41	5.93	5.45	5.15
0.6	5.34	5.06	5.01	5.00
0.5	5.02	5.00	5.00	5.00
0.4	5.00	5.00	5.00	5.00
0.3	5.00	5.00	5.00	5.00
0.2	5.00	5.00	5.00	5.00
0.1	5.00	5.00	5.00	5.00
0.01	5.00	5.00	5.00	5.00

different process settings given in Tables 1-4. We use the moving average sizes of $w = 3$ and 5 with $\beta = 1.5, 2$ for the parameters of the Weibull distribution while the subgroup sizes of $n = 20, 30, 40$ and 50 are considered. The ARL performance of the chart has been computed for different shift levels of $\lambda_1 = \delta\lambda_0$, where δ varies from 1.0 to 0.01.

One levels of the target in-control $ARL_0 = 370$ is considered. The algorithm is given below.

Algorithm 1: Algorithm of Evaluating ARLs.

1) The proposed control statistic is plotted and the run length is computed for a specified a and k.
2) The Step 1 is repeated 10000 times and ARL is computed. If the the computed ARL is equal to the specified value, stop. Otherwise, Step 1 and Step 2 are repeated again with different choice of a and k.
3) Using the determined a and k for a shift process the proposed control statistics are computed and plotted against the same limits as of in control process.
4) The ARL is computed for the shifted process.

From Tables 1-4, we note the following trends in control chart parameters:

1) The value ARL decreases as n increases from 20 to 50.
2) For other fixed parameters, ARL decreases more rapidly as w increases from 3 to 5.
3) The value of ARL increases as the shape parameter of the Weibull distribution increases.

IV. SIMULATION STUDY

In this section, we present the performance of the proposed control chart using a simulated data. The data is generated from the Weibull distribution when $\beta = 2$ and $w = 3$. The first 20 samples of size 20 are generated from an in control process with in-control parameter $\lambda_0 = 1$ and the next 20 observations are generated from the shifted process with parameter $\lambda_1 = 0.8\lambda_0$. The numbers of failures D's are obtained by taking $t_0 = a\mu_0 = 0.1266$. The values of D_i and MA_i are given as follows

D_i: 1, 1, 2, 1, 2, 3, 1, 0, 1, 1, 0, 1, 1, 4, 0, 1, 1, 2, 2, 0, 1, 1, 2, 1, 1, 2, 1, 1, 3, 1, 2, 2, 2, 1, 2, 2, 5, 3, 1, 0

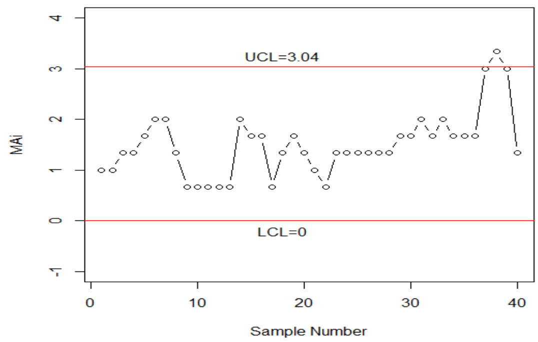

FIGURE 1. The proposed chart for simulated data.

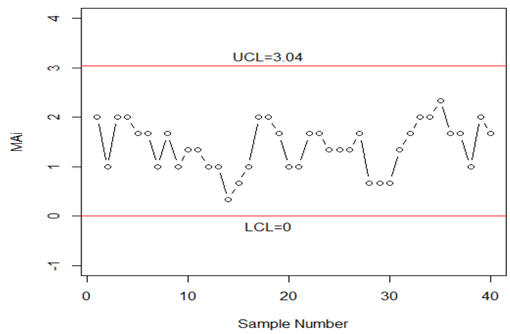

FIGURE 2. The existing chart for simulated data.

MA_i : 1.00, 1.00,1.33,1.33,1.67,2.00,2.00, 1.33, 0.67, 0.67, 0.67, 0.67, 0.67, 2.00, 1.67, 1.67, 0.67, 1.33, 1.67, 1.33, 1.00, 0.67, 1.33, 1.33, 1.33, 1.33, 1.33, 1.33, 1.67, 1.67, 2.00, 1.67, 2.00, 1.67, 1.67, 1.67, 3.00, 3.33, 3.00, 1.3

The values of statistics MA_i are plotted on Figure 1. From this figure, it can be seen that the proposed control chart detects the shift at 36th sample. The values of statistic also plotted for the existing chart proposed by [23, Fig. 2]. From Figure 2, it is evident that the existing control chart does not detect any shift in the process.

TABLE 5. Comparisons of proposed chart having $w = 5$ with existing chart in [22] when, $ARL_0 = 370$ and $\beta = 2$.

	$n = 20$		$n = 30$		$n = 40$		$n = 50$	
	Proposed Chart	Existing chart [23]	Proposed Chart	Existing chart [23]	Proposed Chart	Existing chart [23]	Proposed Chart	Existing chart
δ	ARLs							
1	364.29	375.49	382.26	374.02	366.22	376.43	371.75	368.27
0.99	303.53	333.73	299.82	329.57	307.86	323.66	276.49	321.17
0.95	139.41	208.64	119.79	187.73	121.12	174.89	92.74	162.99
0.92	81.28	146.22	65.35	121.39	61.94	108.13	45.25	93.12
0.9	60.09	115.13	45.51	92.77	41.20	79.43	29.92	67.70
0.85	28.80	65.41	20.65	48.72	17.71	39.43	13.20	31.87
0.8	16.25	38.47	11.51	26.93	9.67	20.21	7.77	16.65
0.7	7.41	14.37	5.93	9.24	5.45	6.74	5.15	5.12
0.6	5.34	6.28	5.06	3.93	5.01	2.82	5.00	2.32
0.5	5.02	3.12	5.00	1.98	5.00	1.51	5.00	1.29

The values of ARLs of the proposed chart with $w = 5$ when $ARL_0 = 370$ and $\beta = 2$ and those from the chart proposed by [22] are also placed in Table 5. From Table 5, it can be noted that the proposed control chart provides smaller ARLs as compared to the existing chart proposed by [22] for all levels of δ. So, the proposed control chart is more efficient than [22] chart in terms of ARLs.

V. INDUSTRIAL EXAMPLE

In this section, the application of the proposed control chart will be discussed using real data. This data represents the time in months until service is needed to a particular sub-system of a passenger car from a leading auto-mobile manufacturing company. The similar data has been used by [22]. The data is known to follow the Weibull distribution with $\beta = 2$. It is assumed here that $w = 3$, $n = 40$, $ARL_0 = 370$ and a =0.29. The values of D_i and MA_i are given as follows

D_i: 1,2,1,1,1,0,3,1,1,2,1,1,1,2,1,1,1,1, 1, 2, 1, 0, 3, 3, 2, 2, 1, 0, 3,0,1,0,4,0,2,3,1,0,1,2

MA_i:1,2,1.33,1.33,1,0.67,1.33,1.67,1.33,1.33,1.33,1,1.33, 1.33,1.33,1,1,1,1.33,1.33,1,1.33,2,2.67,2.33,1.67,1,1.33,1.33, 0.33,1.67,1.33,2,1.67,2,1.33,0.67,1.

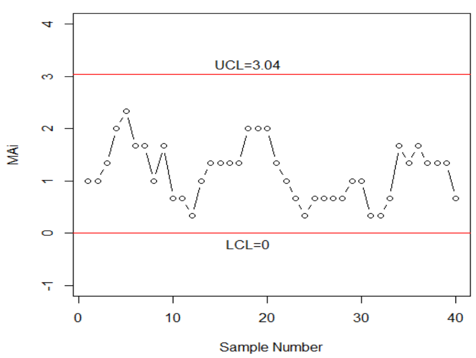

FIGURE 3. The values of MA_i are plotted on chart.

The values of MA_i are plotted on chart in Figure 3. From Figure 3, the process seems to be in control although it can be seen that several values of statistic are near to LCL.

VI. CONCLUSION

A moving average control chart has been suggested for monitoring the number of failures under a time-truncated life test when the life of an item follows the Weibull distribution. The control chart coefficients as well as the test time constant have been determined for different process parameter settings. It has been observed that the proposed control chart will be an efficient addition to the toolkit of the quality control practitioners. The proposed control chart scheme can be extended to some other non-normal distributions. The proposed control chart using EWMA such as in [29] and CUSUM such as in [30] can be considered as future research. The proposed chart can be extended for some other distributions as future research.

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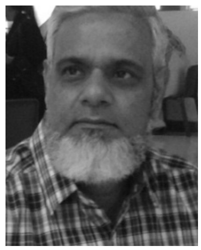
SAEED AHMAD DOBBAH ALGHAMDI received the master's degree from the University of Arkansas, USA, and the Ph.D. degree in statistics from the University of Southampton, U.K. He is currently with the Department of Statistics, Faculty of Science, King Abdulaziz University. His scientific and research interests revolve around the theory of reliability and its applications, biostatistics, quality control, time series, applied statistics, and statistical performance measurement. He has held a number of administrative positions within the university, the last of which is the position of the Vice-Dean of the English Language Institute for external affairs and served as a Statistical Advisor to a number of governmental and private sectors outside the university. His current interests are concentrated in conducting studies and providing statistical consultations regarding quality and reliability, surveys, human capital, and statistical performance measurement.

MUHAMMAD ASLAM received the M.Sc. degree in statistics from Government College University, Lahore, with a Chief Minister of the Punjab Merit Scholarship, in 2004, the M.Phil. degree in statistics from Government College University, with the Governor of the Punjab Merit Scholarship, in 2006, and the Ph.D. degree in statistics from the National College of Business Administration and Economics, Lahore, under the kind supervision of Prof. Dr. M. Ahmad, in 2010. He was a Lecturer of Statistics with the Edge College System International from 2003 to 2006. He was also a Research Assistant with the Department of Statistics, Government College University, from 2006 to 2008. Then, he joined the Forman Christian College University as a Lecturer in 2009, where he was an Assistant Professor from 2010 to 2012. He was with the Department of Statistics as an Associate Professor from 2012 to 2014. He is currently an Associate Professor of Statistics with the Department of Statistics, King Abdulaziz University, Jeddah, Saudi Arabia. He has authored over 220 research papers in national and international journals, including for example, IEEE Access, the *Journal of Applied Statistics*, the *European Journal of Operation Research*, the *Journal of the Operational Research Society*, *Applied Mathematical Modeling*, the *International Journal of Advanced Manufacturer Technology*, *Communications in Statistics*, the *Journal of Testing and Evaluation*, and the *Pakistan Journal of Statistics*. He is the Author of one book published in Germany. He has also been an HEC approved Ph.D. Supervisor since 2011. He supervised five Ph.D. thesis, over 25 M.Phil. theses, and three M.Sc. theses. He is currently supervising two Ph.D. theses and more than five M.Phil. theses in statistics. His areas of interest include reliability, decision trees, industrial statistics, acceptance sampling, rank set sampling, and applied statistics. He reviewed over 70 research papers for various well reputed international journals. He is a member of Editorial Board of the *Electronic Journal of Applied Statistical Analysis*, the *Asian Journal of Applied Science and Technology*, and the *Pakistan Journal of Commence and Social Sciences*. He is also a member of the Islamic Countries Society of Statistical Sciences. He received the meritorious services award in research from the National College of Business Administration and Economics Lahore in 2011 and the Research Productivity Award for the year 2012 by the Pakistan Council for Science and Technology. His name listed at the Second Position among Statistician in the Directory of Productivity Scientists of Pakistan 2013. His name listed at the First Position among Statistician in the Directory of Productivity Scientists of Pakistan in 2014. He got 371 positions in the list of top 2210 profiles of Scientist of Saudi Institutions 2016. He is selected for Innovative Academic Research and the Dedicated Faculty Award 2017 by SPE, Malaysia.



KHUSHNOOR KHAN received the M.S. degree (Hons.) in statistics from the University of the Punjab in 1987, and the MBA degree (executive) from Gomal University in 2010. He has served as a Captain of the Pakistan Army and as a Statistics Instructor with the Pakistan Military Academy, Kakul, Pakistan, from 1990 to 1993. From 2006 to 2010, he was an Adjunct Faculty Member with the Business Administration of International Islamic University, Riphah University, Fatima Jinnah Women University, and the Centre for Advanced Studies in Engineering, Islamabad. He has been a Lecturer with the Department of Statistics, King Abdulaziz University, since 2011. His field of interest is in applied statistics with a special inclination on contingency framework in industrial psychology and education.

CHI-HYUCK JUN (M'91) was born in Seoul, South Korea, in 1954. He received the B.S. degree in mineral and petroleum engineering from Seoul National University in 1977, the M.S. degree in industrial engineering from KAIST in 1979, and the Ph.D. degree in operations research from the University of California at Berkeley, Berkeley, in 1986. Since 1987, he has been with the Department of Industrial and Management Engineering, POSTECH, where he is currently a Professor and the Department Head. He is interested in reliability and quality analysis, and data mining techniques. He is a member of INFORMS and ASQ.