IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received June 15, 2020, accepted July 3, 2020, date of publication July 8, 2020, date of current version July 21, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.3007949

Induced Norm-Based Analysis for Computed Torque Control of Robot Systems

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This work was supported in part by the National Research Foundation of Korea (NRF) funded by the Korea Government (MSIT) under Grant 2020R1G1A1003506 and Grant 2019R1A4A1029003.

ABSTRACT This paper is concerned with providing a new quantitative analysis method of computed torque control of robot systems by using three types of induced norm. This paper first considers design of a computed torque controller to achieve a trajectory tracking performance of a robot manipulator. Taking into account of the effects of unknown elements on the tracking performance, this paper next divides the unknown elements into model uncertainty and disturbance, and introduces various closed-loop representations of robot systems consisting of robot manipulators together with computed torque controllers and unknown elements. This paper further derives a readily applicable robust stability condition for the model uncertainty by using two types of induced norm from L_2 to L_2 and from L_∞ to L_∞. Regarding a performance analysis for the disturbance, this paper also proposes to take the induced norm from L_2 to L_∞, by which the relation between the maximum tracking errors caused by the disturbance and the corresponding parameters of the computed torque controllers are dealt with. Finally, this paper gives some experiments to validate the effectiveness of the performance analysis methods based on the L_∞/L_2-induced norm.

INDEX TERMS Control system analysis, manipulators, performance analysis, robot control.

I. INTRODUCTION

The tracking problem for a given trajectory has been regarded as one of the most important issues in robot systems. To achieve desired tracking performances such as asymptotic stability, minimization of tracking errors, and so on, a number of control approaches have been deeply discussed in [1]–[4], and these approaches considered could be also classified as follows:

1) Model independent method: This method determines the corresponding control parameters independently of the numerical information of the system models for robot systems.
2) Model dependent method: The corresponding control parameters are generally obtained by using detailed system information of robot systems.

One of the most representative schemes in the former method is the proportional-integral-derivative (PID) control, and various stability concepts corresponding to the PID control of robot systems have been actively studied in the literature. For robot systems operated with a simple PID controller, more precisely, the concepts of global asymptotic stability (GAS) and input-to-state stability (ISS) are dealt with in [5]–[7] and [8], [9], respectively, where the GAS does not consider external inputs while the ISS takes into account of exogenous disturbances. However, it is quite difficult to form a quantitative analysis for the PID control of robot systems because these stability concepts are proved by using the general properties of Lagrangian equations without considering detailed numerical information of the dynamics of robot systems.

As one of the most useful schemes in the model dependent method, on the other hand, the computed torque control has been introduced in [10]–[12]. It is required to explicitly computing system models when we take the computed torque control of robot systems, and this allows us to derive a closed-loop representation of the dynamics of the robot systems. In other words, the dynamics of robot systems can be described by a decoupled and linear time-invariant (LTI) equation via a linearization scheme of the computed torque control treatment. Taking into account of the advantages of the decoupled and LTI nature, a number of schemes to the

The associate editor coordinating the review of this manuscript and approving it for publication was Luigi Biagiotti.

computed torque control with respect to theoretical contributions for robot systems have been discussed in [13]–[19]. Regarding the effectiveness and validity of the computed torque control in practical senses, various experimental studies have been also conducted in [20], [21].

Even though it might be generally conclude that the model dependent method of the computed torque control could lead to more theoretically thorough arguments than the model independent method of the PID control in robot systems since the former method involves computing accurate system models and employs the model information in the relevant controller synthesis, such a computation in robot systems is usually quite difficult because of unknown elements such as model uncertainties and external disturbances, which are often occurred in a number of real robot systems. In this regard, the treatment of unknown elements in robot systems could play an important role in establishing a theoretical validity of the computed torque control, and this treatment can be also regarded as the robust or optimal control problem, which have been actively dealt with in the field of control theory (see [22] for details).

In connection with this, based on the arguments of the Lyapunov methods [23], [24], an advanced issue on the robust stability of robot systems operated with the computed torque controllers has been studied deeply in [25]–[27]. The corresponding closed-loop systems connected by robot systems together with the computed torque controllers are shown in these studies to be robustly stable for unknown elements, which are assumed to be bounded in a norm sense. Here, the associated Lyapunov functions should be defined according to the assumptions of some norm constraints on the unknown elements. Even though this approach is quite useful to guarantee the robust stability of the closed-loop systems, it is difficult to construct a quantitative performance measure relevant to the computed torque control for unknown elements by using the arguments in [25]–[27].

With this in mind, this paper introduces another type of robust stability analysis by using the concept of small-gain theorem and proposes a new performance measure with respect to unknown elements in a quantitative fashion. To this end, the unknown elements are divided into model uncertainty and disturbance, and robust stability conditions for the model uncertainty are formulated by using two types of induced norm from L_2 to L_2 and from L_∞ to L_∞ (which are denoted by the L_2/L_2-induced norm and the L_∞/L_∞-induced norm, respectively, for simplicity). Here, it would be worthwhile to note that the unknown elements considered in this paper could be naturally assumed to have finite energy or be bounded persistent. In this sense, taking the L_2/L_2-induced norm and the L_∞/L_∞-induced norm is quite meaningful since the L_2 norm of a signal could describe the energy of the signal while the L_∞ norm of a signal could represent the maximum magnitude of the signal. Subsequently, the relevant performance specification on the regulated output is considered in terms of time-domain bounds, and the regulated output is defined as a sort of weighting function of trajectory tracking errors. Indeed, the relation between the maximum magnitude of the regulated output with the energy of the disturbance is concerned with in this paper; the induced norm from L_2 to L_∞ is taken to tackle such a problem.

On the other hand, it should be remarked that even though the L_∞/L_2-induced norm and the L_∞/L_∞-induced norm have been considered for continuous-time systems [28]–[31] and sampled-data systems [32]–[37], their applications to the performance analyses for the computed torque control of robot systems are studied in this paper for the first time. Finally, we remark that the arguments in this paper are significant extensions of the authors' conference study [38], in which the theoretical results, especially for the robust stability analysis, are omitted and the only simple interpretations about the experiment results are provided.

The notations used in this paper are as follows. We use the notations $\mathbb{R}^\nu$, $d_{max}(\cdot)$ and $tr(\cdot)$ to mean the set of ν-dimensional real numbers, the maximum diagonal entry and the trace of a real symmetric matrix, respectively. The notations $\delta(\theta)$ and e_i imply the impulse function occurring at $\theta = 0$ and the ith vector in the natural basis for $\mathbb{R}^\nu$, respectively. We use the notation $|\cdot|_2$ to mean either the 2-norm of a finite-dimensional vector, i.e.,

$$|x|_2 := (x^T x)^{1/2} \tag{1}$$

or the 2-induced norm of a finite-dimensional matrix, i.e.,

$$|A|_2 := \sup_{|x|_2 \neq 0} \frac{|Ax|_2}{|x|_2} \tag{2}$$

The notation $|\cdot|_\infty$ is used to imply either the ∞-norm of a finite-dimensional vector, i.e.,

$$|x|_\infty := \max_i |x_i| \tag{3}$$

or the ∞-induced norm of a finite-dimensional matrix, i.e.,

$$|A|_\infty := \sup_{|x|_\infty \neq 0} \frac{|Ax|_\infty}{|x|_\infty} \tag{4}$$

Consequently, the notations $\|\cdot\|_2$ and $\|\cdot\|_\infty$ imply the L_2 and L_∞ norms of a vector function, respectively, i.e.,

$$\|f(\cdot)\|_2 := \left(\int_0^\infty |w(t)|_2^2 dt \right)^{1/2} \tag{5}$$

$$\|f(\cdot)\|_\infty := \operatorname*{ess\,sup}_{0 \le t < \infty} |f(t)|_\infty \tag{6}$$

The induced norms from L_2 to L_2, L_2 to L_∞ and from L_∞ to L_∞ of a system are denoted by $\|\cdot\|_{2/2}$, $\|\cdot\|_{\infty/2}$ and $\|\cdot\|_{\infty/\infty}$, respectively, and we call them the L_2/L_2-induced norm, L_∞/L_2-induced norm and L_∞/L_∞-induced norm, respectively, for simplicity.

This paper is organized as follows. The dynamics of robot systems together with their computed torque control approach are introduced in Section II. The new type of robust stability analysis for computed torque control of robot systems based on small-gain theorem together with the treatment of the L_∞/L_2-induced and L_∞/L_∞-induced norms as the performance measures are provided in Section III. Some experiment results are given in Section IV to demonstrate the effectiveness and validity of the induced norm-based performance analysis methods for robot systems. Finally, we give the concluding remarks in Section V.

II. COMPUTED TORQUE CONTROL OF ROBOT SYSTEMS

This section introduces the dynamics of robot systems together with some issues on their computed torque control approach. First of all, we note that robot systems can be generally described by the Lagrangian equation

$$M(q(t))\ddot{q}(t) + C(q(t), \dot{q}(t))\dot{q}(t) + G(q(t)) = \tau(t) + \tau_d(t) \tag{7}$$

where $M(q(t)) \in \mathbb{R}^{n\times n}$, $C(q(t), \dot{q}(t)) \in \mathbb{R}^{n\times n}$ and $G(q(t)) \in \mathbb{R}^n$ are the Inertia matrix, Coriolis and centrifugal matrix and gravitational vector, respectively, while $q(t) \in \mathbb{R}^n$, $\tau(t) \in \mathbb{R}^n$ and $\tau_d(t) \in \mathbb{R}^n$ are the generalized coordinate vector, control input torque vector and external unknown torque vector, respectively.

We next consider the tracking problem for the robot systems of (7) given by

$$q(t) \to q_d(t) \quad (t \to \infty) \tag{8}$$

with the desired trajectory $q_d(t) \in \mathbb{R}^n$. In addition, we introduce the following properties of the Lagrangian equation (7) together with the desired trajectory in (7), which play important roles in establishing a theoretical validity for the control methods developed in this paper.

(i) $M(q(t)),\ \forall q(t) \in \mathbb{R}^n$ is a positive definite matrix.

(ii) There exist positive constants $\alpha_*, \alpha^* \in \mathbb{R}$ such that $\alpha_* I_n \le M(q(t)) \le \alpha^*,\ \forall q(t) \in \mathbb{R}^n$.

(iii) There exists a positive constant $k_c \in \mathbb{R}$ such that $|C(q(t), x)y|_2 \le k_c|x|_2|y|_2,\ \forall q(t), x, y \in \mathbb{R}^n$.

(iv) There exists a positive constant $k_g \in \mathbb{R}$ such that $|G(q(t))|_2 \le k_g,\ \forall q(t) \in \mathbb{R}^n$.

(v) Assume that the initial value of $G(\cdot)$ is zero, i.e., $G(0) = 0$.

(vi) The signals considered in the robot system are norm-bounded, i.e.,
$\|[q_d^T\ \dot{q}_d^T\ \ddot{q}_d^T]^T\|_\infty < \infty,\ \|[q_d^T\ \dot{q}_d^T\ \ddot{q}_d^T]^T\|_2 < \infty,$
$\|[q^T\ \dot{q}^T\ \ddot{q}^T]^T\|_\infty < \infty,\ \|[q^T\ \dot{q}^T\ \ddot{q}^T]^T\|_2 < \infty$
$\|\tau\|_\infty < \infty,\ \|\tau\|_2 < \infty.$

Here, it is a non-trivial task to derive a closed-loop form associated with the tracking problem of (8) because of the nonlinear and coupled characteristics of (7), and thus we are in a position to derive a closed-loop form associated with the tracking problem with the aforementioned properties in mind. More precisely, the nominal values of the dynamics of (7) are assumed to be readily obtained in a real-time sense and we consider the computed torque control [10]–[12] described by

$$\tau(t) = \hat{M}(q(t))(\ddot{q}_d(t) - u(t)) + \hat{C}(q(t), \dot{q}(t))\dot{q}(t) + \hat{G}(q(t)) \tag{9}$$

where the notation $\hat{(\cdot)}$ means the nominal value of $(\cdot)$. By substituting (9) into (7), we can obtain

$$\begin{aligned} M(q(t))\ddot{q}(t) &= \hat{M}(q(t))(\ddot{q}_d(t) - u(t)) \\ &\quad - \tilde{C}(q(t), \dot{q}(t))\dot{q}(t) - \tilde{G}(q(t)) + \tau_d(t) \\ &= M(q(t))(\ddot{q}_d(t) - u(t)) - \tilde{M}(q(t))(\ddot{q}_d(t) - u(t)) \\ &\quad - \tilde{C}(q(t), \dot{q}(t))\dot{q}(t) - \tilde{G}(q(t)) + \tau_d(t) \end{aligned} \tag{10}$$

where $\tilde{M}(q(t)) := M(q(t)) - \hat{M}(q(t))$, $\tilde{C}(q(t), \dot{q}(t)) := C(q(t), \dot{q}(t)) - \hat{C}(q(t), \dot{q}(t))$ and $\tilde{G}(q(t)) := G(q(t)) - \hat{G}(q(t))$ denote the corresponding modeling errors. This further leads to

$$\begin{aligned} \ddot{e}(t) &= u(t) + M^{-1}(q(t))\tilde{M}(q(t))(\ddot{q}_d(t) - u(t)) \\ &\quad + M^{-1}(q(t))\tilde{C}(q(t), \dot{q}(t))\dot{q}(t) \\ &\quad + M^{-1}(q(t))\tilde{G}(q(t)) - M^{-1}(q(t))\tau_d(t) \end{aligned} \tag{11}$$

where $e(t) := q_d(t) - q(t)$ is the trajectory tracking error.

The conventional studies on the computed torque control of robot systems ignore the effects of the modeling errors together with the external disturbance, i.e., it is assumed that $\tilde{M}(q(t)) = 0,\ \tilde{C}(q(t), \dot{q}(t)) = 0,\ \tilde{G}(q(t)) = 0,\ \forall q(t), \dot{q}(t) \in \mathbb{R}^n$ and $\tau_d(t) = 0,\ \forall t \in \mathbb{R}_+$. Such an assumption immediately leads to the following simple linear time-invariant (LTI) representation:

$$\ddot{e}(t) = u(t) \tag{12}$$

Based on (12), the most representative control scheme to u is a sort of proportional-derivative (PD) control described by

$$u(t) = -K_P e(t) - K_D \dot{e}(t) \tag{13}$$

where K_P and K_D are positive diagonal matrices. This immediately leads to

$$\ddot{e}(t) + K_D\dot{e}(t) + K_P e(t) = 0 \tag{14}$$

and it is necessary assumed that all the roots of the corresponding characteristic equation

$$Is^2 + K_D s + K_P = 0 \tag{15}$$

are located in the open left half plane (OLHP). With this necessary condition, we can easily see that $e(t)$ as well as $\dot{e}(t)$ converge to zero as t becomes larger. To summarize, we could guarantee the trajectory tracking performance such as $e(t) \to 0\ (t \to \infty)$ (as well as $\dot{e}(t) = 0\ (t \to \infty)$) by using a simple PD control form of (13) when both the modeling errors and external disturbances do not exist.

However, it is often required to take into account of the effects of model uncertainties and external disturbances since the aforementioned assumptions cannot be generally constructed in a number of real robot systems. To put it another way, because there inevitably exist the modeling errors and external disturbances that usually affect the stability and performance of robot systems, it is quite important to establish a theoretical framework for the treatment of the unknown elements occurred in computed torque control of robot systems. This motivates us to construct new problem definition with the consideration of such unknown elements, and it could be formulated as follows:

(A) **Definition of standard robust control problem:** It is very useful to derive a standard form of robust control problem when we deal with unknown elements occurred in computed torque control of robot systems. If such a standard form is adequately defined, a number of conventional methods of analysis and synthesis for robust control systems might be applied to computed torque control of robot systems.

(B) **Development of quantitative analysis methods:** It should be considered to construct quantitative measures when it is required to evaluate performances for computed torque control of robot systems corresponding to unknown elements. Subsequently, it is also necessary to develop computational schemes which are readily applicable to such quantitative measures.

III. MAIN RESULTS

This section introduces the main results of this paper. To take into account of the effects of unknown elements on stability and performance of computed torque control of robot systems, we first divide the unknown elements into model uncertainty and disturbance. We then derive various closed-loop representations by which the relation between the robot systems together with the model uncertainty and the disturbance can be described by using LTI first-order ordinary differential equations. Based on such representations, we introduce a robust stability condition for model uncertainty of computed torque control of robot systems. Furthermore, we propose to take the L_∞/L_2-induced norm as the performance measure corresponding to a computed torque controller for the disturbance.

A. PROBLEM DESCRIPTION OF A STANDARD ROBUST CONTROL

As a preliminary step to derive an LTI representation of computed torque control of robot systems, we define the disturbance vector $w(t) \in \mathbb{R}^n$ and the model uncertainty element $\Delta(q(t)) \in \mathbb{R}^{n\times n}$ respectively as

$$w(t) := M^{-1}(q(t))(\tilde{M}(q(t))\ddot{q}_d(t) + \tilde{C}(q(t), \dot{q}(t))\dot{q}(t) + \tilde{G}(q(t)) - \tau_d(t)) \quad (16)$$

$$\Delta(q(t)) := -M^{-1}(q(t))\tilde{M}(q(t)) \quad (17)$$

Then, the error dynamics of (11) can be represented by

$$\ddot{e}(t) := (I + \Delta(q(t)))u(t) + w(t) \quad (18)$$

and it immediately follows that

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0 \\ I+\Delta \end{bmatrix} u + \begin{bmatrix} 0 \\ I \end{bmatrix} w \quad (19)$$

We next consider the regulated output which depends on the desired performance specifications for the trajectory tracking problem, and it could be defined as a function of the trajectory tracking error such that

$$z(t) := C_p e(t) + C_d \dot{e}(t) \quad (20)$$

where C_p and C_d are weighting constant matrices to be suitably determined by the user. Furthermore, it is assumed that both $e(t)$ and $\dot{e}(t)$ can be directly measured in a real-time sense throughout the paper. By combining the above procedures, let us introduce the continuous-time LTI generalized plant P_Δ shown in Fig. 1, i.e., P_Δ is given by

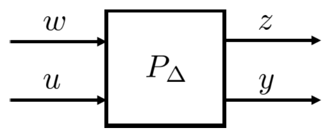


FIGURE 1. Generalized plant with uncertainty for computed torque control of robot systems.

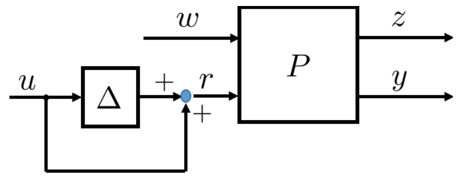


FIGURE 2. Nominal plant with multiplicative input uncertainty.

$$P_\Delta : \begin{cases} \dot{x} = Ax + Bw + B_\Delta u \\ z = Cx \\ y = x \end{cases} \quad (21)$$

where

$$A := \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \quad B := \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad B_\Delta := \begin{bmatrix} 0 \\ I+\Delta \end{bmatrix}, \quad C := \begin{bmatrix} C_p & C_d \end{bmatrix} \quad (22)$$

and $x(t) := [e^T(t)\ \dot{e}^T(t)]^T \in \mathbb{R}^{2n}$ is the state vector, $w(t) \in \mathbb{R}^n$ is the disturbance vector, $u(t) \in \mathbb{R}^n$ is the control input vector, $y(t) \in \mathbb{R}^n$ is the measured output vector and $z(t) \in \mathbb{R}^{n_z}$ is the regulated output vector.

Here, it is a nontrivial task to derive a meaningful control scheme in the treatment of P_Δ since it involves the model uncertainty element Δ. Hence, we consider the continuous-time LTI nominal plant P given by

$$P : \begin{cases} \dot{x} = Ax + Bw + Br \\ z = Cx \\ y = x \end{cases} \quad (23)$$

and represent P_Δ by the control system with multiplicative input uncertainty as shown in Fig. 2 (with $r := (I + \Delta)u$). In other words, the relation from (w, u) to (z, y) of P_Δ in Fig. 1 are the same as that of the nominal plant with multiplicative input uncertainty in Fig. 2.

It should be remarked that, more importantly, the representation of the nominal plant with multiplicative input uncertainty as shown in Fig. 2 could transform the problem for

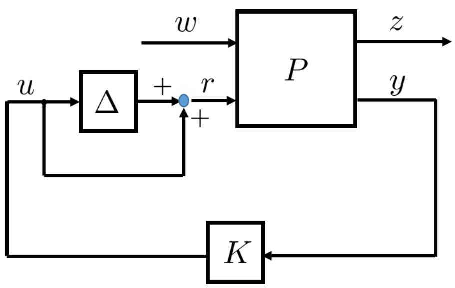


FIGURE 3. Closed-loop system Σ_Δ.

considering Δ occurred in computed torque control of robot systems into the following standard robust control problems associated with stability and performance.

1) **Robust stabilization problem:** It is naturally required to establish stability for unknown elements occurred in computed torque control of robot systems. To this end, the corresponding robust stability condition should be first introduced. Based on such a condition, it could be determined whether or not control laws proposed by the users can stabilize the associated closed-loop system consisting of the control laws and robot systems for the unknown elements.
2) **Performance analysis problem:** In addition to the aforementioned robust stability condition, it is quite useful to construct quantitative performance measures together with their computational methods for the unknown elements in computed torque control of robot systems. To put it another way, such measures could evaluate the performances of control laws proposed by the users for the unknown elements in a numerical/practical sense.

B. ROBUST STABILITY ANALYSIS

This section deals with constructing a simple method of robust stability analysis for computed torque control of robot systems described by the representation as shown in Fig. 2. As a preliminary step to derive a stability condition, we consider an LTI feedback controller for P which stabilizes the corresponding closed-loop system becomes stable because the nominal plant P given by (23) is unstable.

Let K be such a stabilizing controller described by

$$K : \begin{cases} \dot{x}_K = A_K x_K + B_K y \\ u = C_K x_K + D_K y \end{cases} \tag{24}$$

and consider the corresponding closed-loop system Σ_Δ as shown in Fig. 3. Here, we denote the closed-loop system obtained by connecting P and K by Σ (with $\Delta = 0$), and it should be assumed in advance that Σ is stable when we tackle the problem of robust stability analysis for Σ_Δ. To put it another way, the controller parameters (A_K, B_K, C_K, D_K) should be selected to make the state-space equation of Σ described by

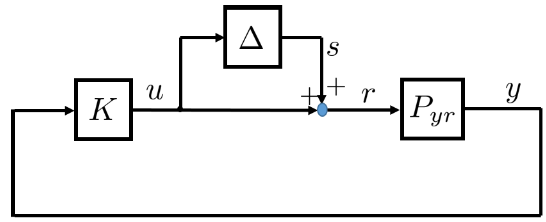


FIGURE 4. Simplified version of Σ_Δ.

$$\Sigma : \begin{cases} \begin{bmatrix} \dot{x} \\ \dot{x}_K \end{bmatrix} = \begin{bmatrix} A + BD_K & BC_K \\ B_K & A_K \end{bmatrix} \begin{bmatrix} x \\ x_K \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} w \\ z = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_K \end{bmatrix} \end{cases} \tag{25}$$

stable (i.e., all the eigenvalues of $\begin{bmatrix} A + BD_K & BC_K \\ B_K & A_K \end{bmatrix}$ are located in the OLHP). With this assumption, we introduce the simplified version of P by ignoring both the disturbance w and the regulated output z, and denote it by P_{yr}. This together with defining $s := \Delta u$ also leads to the simplified version of the closed-loop system Σ_Δ as shown in Fig. 4. Such a simplified version can be also equivalently converted to the standard robust control system as shown in Fig. 5 by letting Σ_{us} be the closed-loop system from s to u obtained by connecting P_{yr} to K in Fig. 4.

Remark 1: Even though it is sufficient to consider the effect of s on u in the simplified version of Σ_Δ shown in Fig. 4 to construct the associated robust stability condition, we will return to the general case with the consideration of w and z for the corresponding performance analysis in the following subsection.

Based on the standard representation shown in Fig. 5, we are led to the following theorem relevant to the robust stability of the closed-loop system Σ_Δ.

Theorem 1: The closed-loop system Σ_Δ is stable for all Δ such that $\|\Delta\|_{2/2} \leq \gamma_2$ if and only if

$$\|\Sigma_{us}\|_{2/2} < 1/\gamma_2 \tag{26}$$

and is also stable for all Δ such that $\|\Delta\|_{\infty/\infty} \leq \gamma_\infty$ if and only if

$$\|\Sigma_{us}\|_{\infty/\infty} < 1/\gamma_\infty \tag{27}$$

We omit the proof of this theorem because it could be easily followed by using the arguments of small-gain theorem [22], [39]. It should be remarked that the main contribution of this subsection is to derive Theorem 1 which constructs the new concept of robust stability analysis for computed torque control of robot systems by using the standard form of small-gain theorem for the first time. To put it another way, the assertions of Theorem 1 are intrinsically different to the conventional studies [25]–[27] on robust stability analysis for computed torque control of robot systems, in which

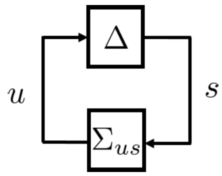


FIGURE 5. Standard robust control system.

one should adequately define the corresponding Lyapunov functions. In contrast, the arguments of Theorem 1 could be readily applied to the real robot systems equipped with computed torque controllers because both the L_2/L_2 and L_∞/L_∞-induced norms can be easily obtained by using the arguments in [22] and [40], respectively.

Regarding the norm boundness of Δ considered in Theorem 1, on the other hand, we note the definition of Δ in (17) together with the properties (i) and (ii) introduced in Section II. In other words, it could be readily established from these arguments that both $|\Delta(q(t))|_2$ and $|\Delta(q(t))|_\infty$ are bounded for all $q(t) \in \mathbb{R}^n$ since $M(q(t))$, $\hat{M}(q(t))$ and $M^{-1}(q(t))$ are also bounded for all $q(t) \in \mathbb{R}^n$. Hence, if the matrix $\Delta(q(t))$ is regarded as a mapping from u to s, then it is just a sort of feedthrough term, and thus the L_2/L_2-induced norm $\|\Delta\|_{2/2}$ and the L_∞/L_∞-induced norm $\|\Delta\|_{\infty/\infty}$ are naturally defined as $\sup_{q(t)\in\mathbb{R}^n} |\Delta(q(t))|_2$ and $\sup_{q(t)\in\mathbb{R}^n} |\Delta(q(t))|_\infty$, respectively. Thus, we can see that the norm boundness of Δ assumed in Theorem 1 is valid in both theoretical and practical senses.

C. PERFORMANCE ANALYSIS

In contrast to the preceding subsection associated with robust stability analysis, this subsection considers quantitative performance analysis for computed torque control of robot systems.

To this end, we first return to the general case of the closed-loop system Σ_Δ shown in Fig. 3 and revisit the state-space equation of Σ given by (25). We represent (25) by

$$\Sigma : \begin{cases} \dot{\xi} = \mathcal{A}\xi + \mathcal{B}w \\ z = \mathcal{C}\xi \end{cases} \tag{28}$$

where

$$\mathcal{A} := \begin{bmatrix} A + BD_K & BC_K \\ B_K & A_K \end{bmatrix}, \quad \mathcal{B} := \begin{bmatrix} B \\ 0 \end{bmatrix}, \ \mathcal{C} := \begin{bmatrix} C & 0 \end{bmatrix} \tag{29}$$

with $\xi := \begin{bmatrix} x^T & x_K^T \end{bmatrix}^T$.

As mentioned in the preceding subsection, the first consideration in selecting the controller parameters A_K, B_K, C_K and D_K is to make $\mathcal{A}$ stable, i.e., all the eigenvalues of $\mathcal{A}$ should be located in the OLHP. In addition, they have been also determined corresponding to desired natural frequencies and damping ratios relevant to the performance specifications.

However, this approach does not fit into dealing with the effect of the disturbance w on the regulated output z since it is practically difficult to predetermine frequency bands for w and z. More importantly, both natural frequency and damping ratio are not suitable for characterizing the regulated output z when we are interested in reducing the maximum magnitude of z in the time-domain.

With this in mind, the aim of this subsection is to introduce a quantitative performance measure for computed torque control of robot systems with the consideration of reducing the maximum magnitude of the regulated output. As a preliminary step to propose an associated performance measure, we note the characteristics of w from (16) together with the properties (i)–(vi) introduced in Section II. In other words, we can see from these properties that

$$\|w\|_2 < \infty, \quad \|w\|_\infty < \infty \tag{30}$$

To summarize, because both the L_∞ norm $\|w\|_\infty$ and the L_2 norm $\|w\|_2$ are bounded, we can take both the L_∞/L_∞-induced norm and the L_∞/L_2-induced norm when we consider the relation between the maximum magnitude of z with w. However, this paper is mainly concerned with taking the L_∞/L_2-induced norm as the performance measure rather than the L_∞/L_∞-induced norm. This is because the L_2 space is a Hilbert space while the L_∞ space is a Banach space, and thus a number of optimization schemes can be applied to the L_2 space (but almost of them cannot be usually applied to the L_∞ space). Indeed, the L_∞/L_∞-induced norm cannot be exactly computed and we should consider an approximation approach, while the L_∞/L_2-induced norm could be obtained in an analytic fashion.

In this sense, we are in a position to take the L_∞/L_2-induced norm as such a quantitative performance measure, and briefly introduce its computation method together with another interpretation.

We first note from (28) that the input-output behavior of Σ can be given by the convolution integral

$$\begin{aligned} z(t) &= \int_0^t \mathcal{C}\exp(\mathcal{A}(t-\theta))\mathcal{B}w(\theta)d\theta \\ &=: (\mathcal{F}w)(t) \quad (0 \le t < \infty) \end{aligned} \tag{31}$$

where $\mathcal{F}$ can be regarded as an operator from L_2 to L_∞. Because $\mathcal{F}$ is a linear operator and $(\mathcal{F}w)(\cdot)$ is a continuous function, it immediately follows that

$$\begin{aligned} \|\mathcal{F}\|_{\infty/2} &:= \sup_{\|w\|_2\neq 0} \frac{\|\mathcal{F}w\|_\infty}{\|w\|_2} = \sup_{\|w\|_2=1} \|\mathcal{F}w\|_\infty \\ &= \sup_{\|w\|_2=1} \sup_t |\mathcal{F}w(t)|_\infty \end{aligned} \tag{32}$$

This can be further represented by

$$\begin{aligned} \|\mathcal{F}\|_{\infty/2} &:= \sup_t \sup_{\|w\|_2=1} \left| \int_0^t \mathcal{C}\exp(\mathcal{A}(t-\theta))\mathcal{B}w(\theta)d\theta \right|_\infty \\ &= \lim_{t\to\infty} \sup_{\|w\|_2=1} \left| \int_0^t \mathcal{C}\exp(\mathcal{A}(t-\theta))\mathcal{B}w(\theta)d\theta \right|_\infty \\ &= \sup_{\|w\|_2=1} \left| \int_0^\infty \mathcal{C}\exp(\mathcal{A}\theta)\mathcal{B}w(\theta)d\theta \right|_\infty \end{aligned} \tag{33}$$

The last term of (33) can be explicitly obtained by using the continuous-time Cauchy-Schwartz inequality, with the vector-valued functions f_1 and f_2, described by

$$\left(\int_0^t f_1^T(\theta)f_2(\theta)d\theta\right)^2 \leq \int_0^t |f_1(\theta)|_2^2 d\theta \cdot \int_0^t |f_2(\theta)|_2^2 d\theta \quad (34)$$

where the equality holds if and only if $f_1(\theta) = \lambda f_2(\theta)$ ($\forall\theta \in [0, t]$) for a constant λ. By using this inequality, we have that

$$\|\mathcal{F}\|_{\infty/2} = \max_{1\leq i\leq n} \left(\int_0^\infty \mathcal{C}_i \exp(\mathcal{A}\theta)\mathcal{B}\mathcal{B}^T \exp(\mathcal{A}^T\theta)\mathcal{C}_i^T d\theta\right)^{1/2} \quad (35)$$

because

$$\left(\int_0^\infty \mathcal{C}_i \exp(\mathcal{A}\theta)\mathcal{B}w(\theta)d\theta\right)^2 \leq \|\mathcal{B}^T \exp(\mathcal{A}^T(\cdot))\mathcal{C}_i^T\|_2^2 \cdot \|w(\cdot)\|_2^2 \quad (36)$$

where $\mathcal{C}_i$ ($1 \leq i \leq n$) denotes the ith row of $\mathcal{C}$. Here, the integral in the right-hand-side of (35) could be obtained by solving the continuous-time Lyapunov equation

$$\mathcal{A}\mathcal{P} + \mathcal{P}\mathcal{A}^T + \mathcal{B}\mathcal{B}^T = 0 \quad (37)$$

Hence, we obtain

$$\|\mathcal{F}\|_{\infty/2} = \max_{1\leq i\leq n} (\mathcal{C}_i\mathcal{P}\mathcal{C}_i^T) = d_{max}^{1/2}(\mathcal{C}\mathcal{P}\mathcal{C}^T) \quad (38)$$

Then, we are led to the following results.

Theorem 2: The L_∞/L_2-induced norm $\|\mathcal{F}\|_{\infty/2}$ relevant to (28) coincides with $d_{max}^{1/2}(\mathcal{C}\mathcal{P}\mathcal{C}^T)$.

Corollary 1: Let the L_2 norm $\|w\|_2$ is bounded by ρ_2, i.e., $\|w\|_2 \leq \rho_2$. Then, it readily follows that

$$\|z\|_\infty \leq \|\mathcal{F}\|_{\infty/2} \cdot \rho_2 = d_{max}^{1/2}(\mathcal{C}\mathcal{P}\mathcal{C}^T) \cdot \rho_2 \quad (39)$$

Theorem 2 clearly implies that the L_∞/L_2-induced norm $\|\mathcal{F}\|_{\infty/2}$ can be explicitly and easily obtained through the continuous-time Lyapunov equation, while Corollary 1 is a simple consequence of Theorem 2 and the property of the L_∞/L_2-induced norm. It could be expected from this corollary that the maximum magnitude of the regulated output z decreases at a no smaller rate than $1/d_{max}^{1/2}(\mathcal{C}\mathcal{P}\mathcal{C}^T)$. Even though these results are similar to the results in [28], [29] or [32], [33], which are associated with the L_∞/L_2-induced norm of continuous-time LTI systems or sampled-data systems, respectively, they are quite meaningful in the sense that a quantitative performance measure for computed torque controllers against unknown elements is proposed in a readily computable fashion of the L_∞/L_2-induced norm for the first time.

On the other hand, we also provide another interpretation of the L_∞/L_2-induced norm by noting the conventional H_2 norm which corresponds to the L_2 norm of the impulse response. The conventional H_2 norm for Σ, which is denoted by $\|\Sigma\|_{H_2}$, is generally given by

$$\begin{aligned}\|\Sigma\|_{H_2} &:= \left(\sum_{i=1}^n \|\mathcal{F}\delta(\theta)e_i\|_2^2\right)^{1/2} \\ &= \left(\sum_{i=1}^n \|\mathcal{C}\exp(\mathcal{A}(\cdot))\mathcal{B}e_i\|_2^2\right)^{1/2} \\ &= tr^{1/2}\left(\int_0^\infty \mathcal{C}\exp(\mathcal{A}t)\mathcal{B}\mathcal{B}^T\exp(\mathcal{A}^Tt)\mathcal{C}^T dt\right) \\ &= tr^{1/2}\left(\mathcal{C}\mathcal{P}\mathcal{C}^T\right) \quad (40)\end{aligned}$$

If we note that $tr^{1/2}(\mathcal{C}\mathcal{P}\mathcal{C}^T) \geq d_{max}^{1/2}(\mathcal{C}\mathcal{P}\mathcal{C}^T)$, we can readily see from Theorem 2 with (40) that the H_2 norm is not smaller than the L_∞/L_2-induced norm, and they coincide with each other when $n_z = 1$ (i.e., single-output case). This can be summarized as follows.

Theorem 3: The L_∞/L_2-induced norm and the H_2 norm associated with (28) can be described by $d_{max}^{1/2}(\mathcal{C}\mathcal{P}\mathcal{C}^T)$ and $tr^{1/2}(\mathcal{C}\mathcal{P}\mathcal{C}^T)$, respectively. Furthermore, it is obvious that

$$tr^{1/2}(\mathcal{C}\mathcal{P}\mathcal{C}^T) \geq d_{max}^{1/2}(\mathcal{C}\mathcal{P}\mathcal{C}^T) \quad (41)$$

and they coincided with each other for single-output case (i.e., $n_z = 1$).

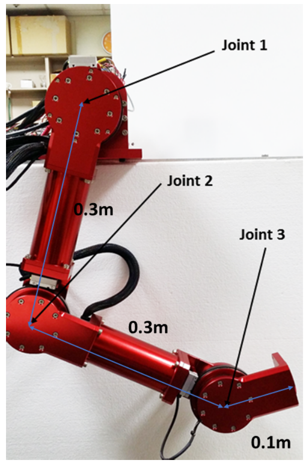


FIGURE 6. 3-DoF robot manipulator.

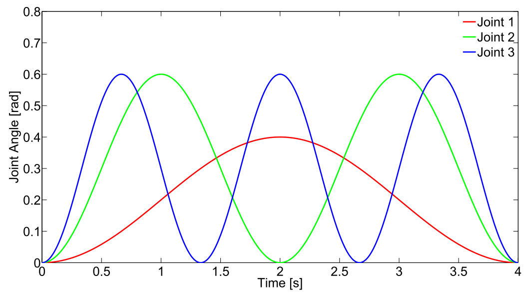
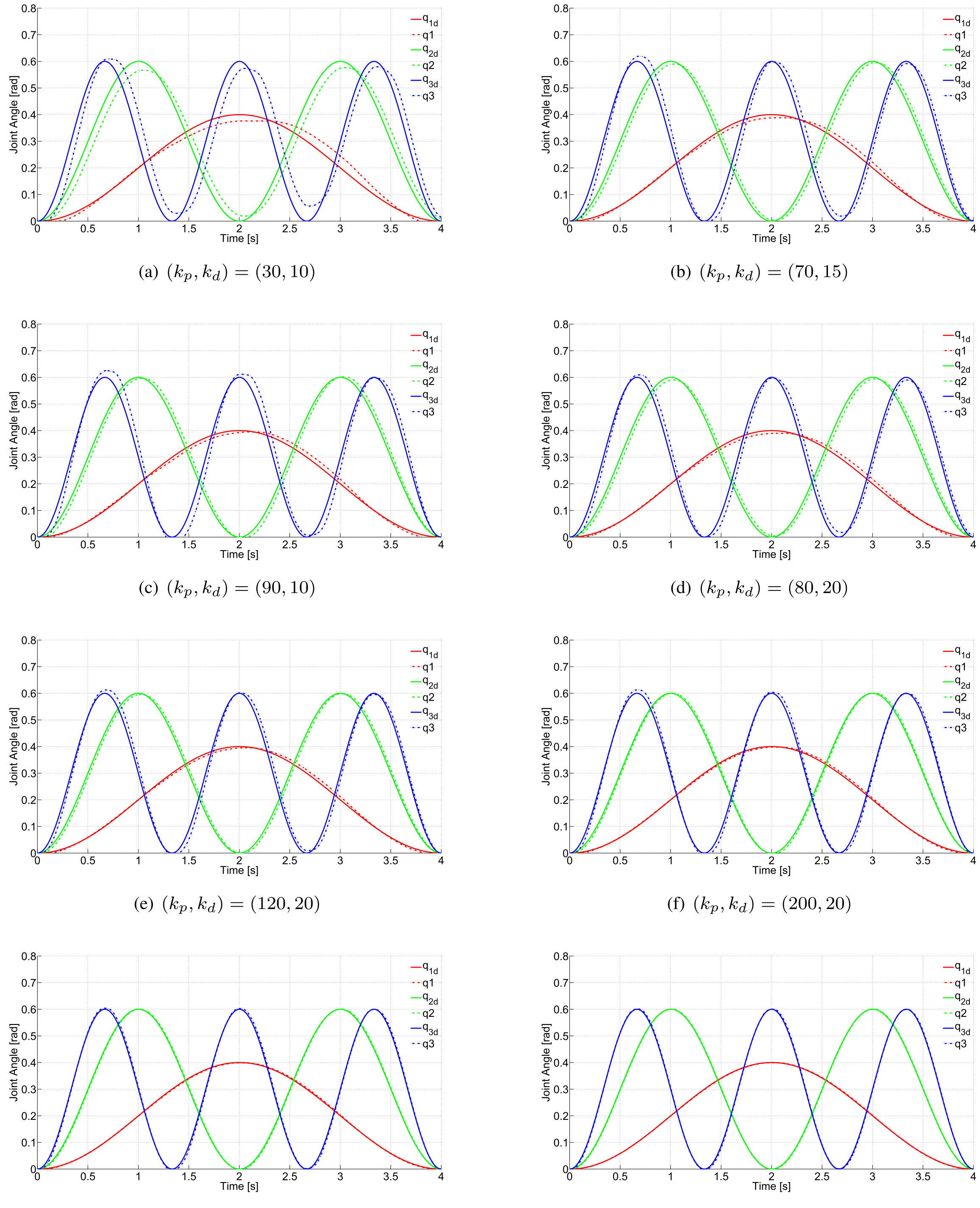


FIGURE 7. Desired trajectories of three joints.



(a) $(k_p, k_d) = (30, 10)$
(b) $(k_p, k_d) = (70, 15)$
(c) $(k_p, k_d) = (90, 10)$
(d) $(k_p, k_d) = (80, 20)$
(e) $(k_p, k_d) = (120, 20)$
(f) $(k_p, k_d) = (200, 20)$
(g) $(k_p, k_d) = (400, 30)$
(h) $(k_p, k_d) = (800, 40)$

FIGURE 8. Comparison between the experiment results for the joint angles $q(t)$ and the reference joint angles $q_d(t)$.

The L_∞/L_2-induced norm can be interpreted as a generalized/specific version of the conventional H_2 norm since they coincide with each other for the single-output case. Even though this induced norm is intrinsically related with a smooth input function (because the domain is the L_2 space), the aforementioned interpretation could say that the L_∞/L_2-induced norm can be an alternative to deal with a non-smooth input function such as impulse disturbances for the single-output case.

IV. EXPERIMENT RESULTS

In this section, we aim at demonstrating the validity of the L_∞/L_2-induced norm as a performance measure for computed torque control of robot systems for disturbances. More

TABLE 1. Experiment results for the L_∞ norm of the regulated output z and the associated L_∞/L_2-induced norm.

(k_p, k_d)	$(30, 10)$	$(70, 15)$	$(90, 10)$	$(80, 20)$
$\|z\|_\infty$	2.0155	1.2329	1.0628	0.8540
$\|\mathcal{F}\|_{\infty/2}$	0.6519	0.4183	0.3748	0.3087
(k_p, k_d)	$(120, 20)$	$(200, 20)$	$(400, 30)$	$(800, 40)$
$\|z\|_\infty$	0.7328	0.5538	0.3570	0.2624
$\|\mathcal{F}\|_{\infty/2}$	0.2681	0.2305	0.1614	0.1266

precisely, the arguments in Theorem 2 and Corollary 1 would be validated through some experiments. The 3-degrees of freedom (3-DoF) robot manipulator as shown in Fig. 6 is employed in this section, where this manipulator consists of three brush-less direct current (BLDC) motors with three linkages whose lengths are 0.3, 0.3 and 0.1 m, respectively. This section deals with the desired trajectories of the three joints as shown in Fig. 7. They are cosine functions, and the angles together with velocities are set to be zero at $t = 0$ [s] and 4 [s].

Subsequently, we take the controller with a static form for simplicity. To put it another way, the controller is assumed to be given by

$$u = D_K y = D_K x = D_K \begin{bmatrix} e \\ \dot{e} \end{bmatrix} =: -\begin{bmatrix} K_p & K_d \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \tag{42}$$

with

$$K_p = \begin{bmatrix} k_p & 0 & 0 \\ 0 & k_p & 0 \\ 0 & 0 & k_p \end{bmatrix}, \quad K_d = \begin{bmatrix} k_d & 0 & 0 \\ 0 & k_d & 0 \\ 0 & 0 & k_d \end{bmatrix} \tag{43}$$

(i.e., the parameters A_K, B_K and C_K in (24) are assumed to be zeros.)

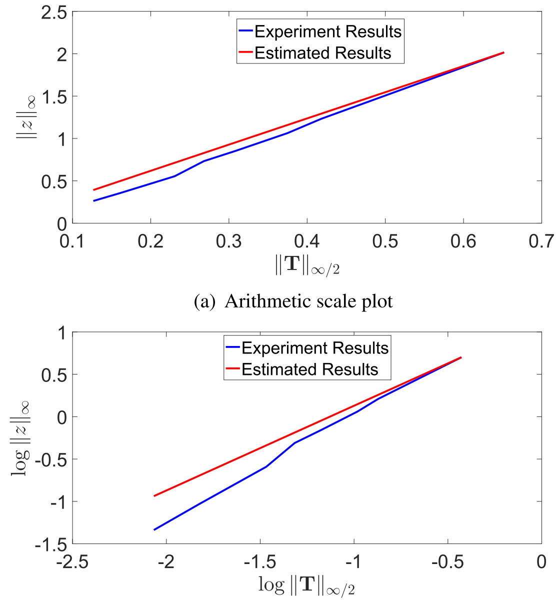
We take various values of the parameters (K_p, K_d) by which the associated L_∞/L_2-induced norm is bounded (i.e., $\mathcal{A}$ has its all eigenvalues in the OLHP). We also consider the regulated output $z = Cx$ with the parameter

$$C = \begin{bmatrix} C_p & C_d \end{bmatrix} = \begin{bmatrix} 15 & 0 & 0 & 1 & 0 & 0 \\ 0 & 15 & 0 & 0 & 1 & 0 \\ 0 & 0 & 15 & 0 & 0 & 1 \end{bmatrix} \tag{44}$$

and observe its L_∞ norm $\|z\|_\infty$.

The corresponding experiment results for the joint angles are shown in Fig. 8. The experiment results for the L_∞ norms of the regulated output z together with the computation results for the L_∞/L_2-induced norm $\|\mathcal{F}\|_{\infty/2}$ of the closed-loop system of (28) are given in Table 1.

It could be observed from Fig. 8 and Table 1 that taking we can achieve better tracking performances by taking the L_∞/L_2-induced norm smaller. We can also confirm from Table 1 that the L_∞ norm of the regulated output z is decreasing as the L_∞/L_2-induced norm $\|\mathcal{F}\|_{\infty/2}$ becomes smaller. These experiment observations clearly imply that taking the L_∞/L_2-induced norm as a performance measure for computed torque controllers against the model uncertainty and disturbance is practically meaningful.



(a) Arithmetic scale plot

(b) Log-Log scale plot

FIGURE 9. Comparison between experiment results and estimated values.

On the other hand, the values of $\|z\|_\infty$ on various values of $\|\mathcal{F}\|_{\infty/2}$ can be also estimated by using the arguments of Corollary 1 once we have a value of $\|z\|_\infty$ on a specific value of $\|\mathcal{F}\|_{\infty/2}$. For example, in this experiment, if we use the values on $(k_p, k_d) = (30, 10)$, then the values of $\|z\|_\infty$ can be estimated as $2.0155 \times (\|\mathcal{F}\|_{\infty/2}/0.6519)$. In this sense, it is worth to compare experiment results with the estimated values obtained by using the arguments in Corollary 1. The results for such a comparison are shown in Fig. 9.

It could be observed from Fig. 9 (a) that the experiment results are not larger than the estimated values under the same $\|\mathcal{F}\|_{\infty/2}$. Indeed, we could confirm from Fig. 9 (b) that $\|z\|_\infty$ relevant to the experiment results is decreasing at no slower convergence rate that relevant to the estimated value since the slope for the experiment results is always more steep than that for the estimated values. These observations clearly demonstrate the validity of the argument in Corollary 1, and thus the L_∞/L_2-induced norm can be effectively used as a performance measure for computed torque controllers.

V. CONCLUDING REMARKS

This paper provided a new quantitative analysis method of computed torque control of robot systems by using three types of induced norm of the L_2/L_2-induced norm, L_∞/L_∞-induced norm and L_∞/L_2-induced norm. To this end, we first dealt with design of computed torque controllers for trajectory tracking problems of robot manipulators. To take into account of the effects of unknown elements occurred in the computed torque control treatment of robot systems, we showed that the unknown elements could be divided into model uncertainty and disturbance. By using this decomposition, we introduced various closed-loop representations of robot systems consisting of robot manipulators together with computed torque controllers and unknown elements. Based on such closed-loop representations, we derived a readily applicable robust stability condition for the model uncertainty by using two induced norms of the L_2/L_2-induced norm and the L_∞/L_∞-induced norm. More precisely, in contrast to the conventional studies on robust stability analysis for computed torque control of robot systems based on Lyapunov theorem, this paper proposed a quantitative robust stability condition by using the arguments of small-gain theorem. To tackle the performance analysis problem of the disturbance, this paper also proposed to take the L_∞/L_2-induced norm, by which the relation between the maximum tracking errors of the regulated output with the disturbance is concerned with. Furthermore, we introduce another interpretation of the L_∞/L_2-induced norm by comparing with the conventional H_2 norm. Indeed, we confirmed from some experiments that taking the L_∞/L_2-induced norm as a performance measure for computed torque controllers against disturbance is practically meaningful. Finally, we believe that the success in this paper in introducing the new performance measure for computed torque controllers based on the L_∞/L_2-induced norm contributes to wider applications of computed torque control approach to a number of practical robot systems.

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