Published for SISSA by Springer

Received: June 3, 2017
Revised: August 16, 2017
Accepted: August 21, 2017
Published: September 6, 2017

JHEP09(2017)022

Notes on worldvolume supersymmetries for D-branes on $\mathrm{AdS}_5\times\mathrm{S}^5$ background

Jaemo Park[a] **and Hyeonjoon Shin**[a,b]

[a] *Department of Physics & Center for Theoretical Physics, POSTECH, Pohang, Gyeongbuk 37673, South Korea*
[b] *Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 37673, South Korea*

E-mail: jaemo@postech.ac.kr, shin67@postech.ac.kr

Abstract: We revisit the 1/2-BPS D-branes on the $\mathrm{AdS}_5\times\mathrm{S}^5$ background. Based only on the classification of 1/2-BPS D-branes obtained by the covariant open string description, we consider various purely static configurations of D-branes without any worldvolume flux on the $\mathrm{AdS}_5\times\mathrm{S}^5$ background. Under the covariant κ symmetry fixing condition, we investigate which part the spacetime supersymmetries is preserved on the D-brane worldvolume and obtain the associated worldvolume supersymmetry transformation rules to leading order in the worldvolume fluctuating fields. It is shown that, for purely static configurations without any worldvolume flux, only the AdS type D-branes, in which the AdS radial direction is one of worldvolume coordinates, are 1/2-BPS.

Keywords: D-branes, Extended Supersymmetry, AdS-CFT Correspondence

ArXiv ePrint: 1705.06887

https://doi.org/10.1007/JHEP09(2017)022

Contents

1 Introduction

D-brane in a given background containing AdS spacetime is an interesting object to explore. It carries the information about the open string sector in the background. In the AdS/CFT correspondence [1–3], a certain D-brane configuration in the background involving the AdS spacetime is related to the defect conformal field theory (dCFT) [4, 5]. For example, a certain D5-brane in the $AdS_5 \times S^5$ background is dual to the three dimensional dCFT of the N=4 SYM theory [5]. In a suitable approximation, D branes on curved spacetime can be described by Born-Infeld type action. The Born-Infeld type action of supersymmetric D-brane has the important symmetry, κ symmetry. By a suitable gauge fixing, one can obtain the supersymmetric worldvolume theory on the Born-Infeld type action of D-brane. In refs. [6, 7], by choosing a static gauge combined with a suitable κ gauge fixing, the supersymmetric worldvolume theory of D-brane in flat space is obtained, with the explicit supersymmetric transformation worked out.

One can certainly adopt the same strategy to D-branes in the background involving the AdS spacetime. In fact, κ symmetric action of D-branes in the $\mathrm{AdS}_5\times\mathrm{S}^5$ background was obtained using supercoset approach. One can guess that again by taking the static gauge with suitable κ gauge fixing condition one can obtain the worldvolume theories of supersymmetric D-branes on the $\mathrm{AdS}_5\times\mathrm{S}^5$ background.[1] However there is a subtlety in this program. In [8], the author considered the D3-brane whose world volume spans the four directions in AdS_5 other than the radial direction but neither the usual covariant gauge fixing condition nor Killing spinor gauge fixing works.[2] Note that the configuration is supersymmetric since the D3 brane of interest is parallel to the D3 branes, whose near horizon geometry turns into $\mathrm{AdS}_5\times\mathrm{S}^5$. Indeed in [9], the D3 brane is shown to satisfy the generalized calibration, hence is supersymmetric.[3] Analogous result is worked out at [11], where they consider M2 brane in $\mathrm{AdS}_4\times\mathrm{S}^7$ where M2 brane is parallel to M2 branes which make AdS geometry. They show that Killing spinor gauge is incompatible with the static gauge of the M2 brane worldvolume action. Given these results, one might wonder if there are some restrictions on the possible supersymmetric worldvolume theory on the AdS spacetime. It turns out that this problem is intimately related to the classification of supersymmetric D-brane embeddings into the AdS spacetime.

In this paper we are looking for the supersymmetric worldvolume theories of 1/2-BPS D-branes on the $\mathrm{AdS}_5\times\mathrm{S}^5$ background. Since the worldvolume supersymmetry is of our concern, it is natural to take the probe brane analysis. In order to study the worldvolume theory of 1/2-BPS D-branes we start from the data obtained from the covariant open string description of supersymmetric D-branes developed in [12, 13]. The covariant description can be applied to any background if the superstring action on it is given and, as the first non-trivial application, has led to the classification of 1/2-BPS D-branes in some plane wave backgrounds [13, 14].[4] As for the $\mathrm{AdS}_5\times\mathrm{S}^5$ background, the classification has been worked out in [17–19] and table 1 shows its result. As a consistency check, we note that the same data listed in table 1 have been also obtained in [20] in the context of pure spinor formalism [21].

The covariant open string description provides us a definite guideline for further study of supersymmetric D-branes, although it gives no more information about D-branes other than the classification data. Starting from the table 1, we consider all possible types of corresponding D-brane configurations and use a suitable static gauge for the D-brane worldvolume diffeomorphism. For the fermionic κ symmetry of the D-brane action, the covariant κ symmetry gauge is adopted. For each of the configurations, we identify the

[1] This will be one more useful example of supersymmetric theories on curved background, which can find the application to the localization of the gauge theory on a curved background.

[2] We think the subtlety arises since the D3 brane whose world volume we consider is parallel to D3 branes, which make the geometry. If we start from intersecting configuration of Dp branes with D3 branes, which are geometrized, such subtlety does not arise. All brane configurations considered in the paper are of this type.

[3] For recent exploration of the generalized calibration in the AdS backgrounds, see for example ref. [10] and references therein.

[4] The IIB plane wave has a connection with the $\mathrm{AdS}_5\times\mathrm{S}^5$ background through the Penrose limit [15]. One may refer a work [16] done in the plane wave background, which may be related to the present one.

	D(-1)	D1	D3	D5	D7	D9
(n,n')	(0,0)	(2,0) (0,2)	(3,1) (1,3)	(4,2) (2,4)	(5,3) (3,5)	—

Table 1. 1/2-BPS D-branes in the $\mathrm{AdS}_5\times\mathrm{S}^5$ background. n (n') represents the number of Neumann directions in AdS_5 (S^5).

worldvolume supersymmetry realized on the D-brane worldvolume and obtain the associated worldvolume supersymmetry transformation rules for the worldvolume fields.

We restrict ourselves to the purely static Lorentzian D-brane configurations without any worldvolume flux. Here the Lorentzian means that the worldvolume time is identified with that of the background spacetime. Thus, the configurations for (0,0) and (0,2) D-branes of table 1 are not considered. As one may realize, there are twelve types of Lorentzian configurations. Six of them correspond to the AdS type D-branes in which the AdS radial direction is one of worldvolume coordinates, and the remaining six are of non-AdS type in which the AdS radial direction is transverse to the D-brane worldvolume. We will treat the AdS and non-AdS type branes separately. We would like to note that the AdS type D-branes may also be considered under the name of AdS embedding of D-branes by taking the near horizon limit of intersecting D3⊥Dp brane configurations in flat spacetime [5]. It turns out that, *for purely static configurations without any worldvolume flux, only AdS type branes admit the supersymmetric worldvolume theories.* Note that the corresponding D-brane configurations are obtained from the supersymmmetric intersecting D-brane configurations after turning D3 branes into AdS geometry [5]. For non-AdS type branes, the analysis suggests that world volume fluxes should be turned on or some motions in transverse directions should be considered to have the supersymmetric world volume theory. Our work suggests that only D-branes tabulated at table 1 admit the supersymmetric worldvolume theory. In order to confirm it, the worldvolume theories of non-AdS type branes should be worked out, which are beyond the scope of this paper.

The organization of this paper is as follows. In section 2, we describe the way of realizing the worldvolume supersymmetry for a given D-brane configuration after reviewing some necessary elements. In section 3, we investigate the worldvolume supersymmetry for the AdS type D-branes. Then the non-AdS branes are considered in section 4. The discussion with our conclusion follows in section 5. Finally, appendix A contains our notation and convention with the expressions of superfields.

2 Generalities

In this section, we briefly review the $\mathrm{AdS}_5\times\mathrm{S}^5$ background with its associated Killing spinor and the symmetries of Dp-brane action. We then describe how to identify the supersymmetry realized on the brane worldvolume for a given brane configuration.

JHEP09(2017)022

2.1 $\mathrm{AdS}_5\times\mathrm{S}^5$ background

In the Poincaré patch coordinates, the metric for the $\mathrm{AdS}_5\times\mathrm{S}^5$ geometry is written as

$$ds^2 = u^2\left[-\left(dx^0\right)^2 + (d\vec{x})^2\right] + \frac{du^2}{u^2} + d\Omega_5^2\,, \tag{2.1}$$

where $(d\vec{x})^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$ and $d\Omega_5^2$ is the metric of S^5 parametrized by five angular coordinates ϕ^α $(\alpha = 1,\ldots,5)$,

$$d\Omega_5^2 = \left(d\phi^1\right)^2 + \sin^2\phi^1\left[\left(d\phi^2\right)^2 + \sin^2\phi^2\left[\left(d\phi^3\right)^2 + \sin^3\phi^2\left[\left(d\phi^4\right)^2 + \sin^4\phi^2\left(d\phi^5\right)^2\right]\right]\right]\,, \tag{2.2}$$

with ranges of $0 \le \phi^1,\phi^2,\phi^3,\phi^4 \le \pi$ and $0 \le \phi^5 \le 2\pi$. Here, we have taken the common radius R of the AdS_5 and S^5 to be one, $R = 1$. The ten dimensional coordinates are equationed as

$$X^\mu = \left\{x^0, x^1, x^2, x^3, u, \phi^1, \ldots, \phi^5\right\} \tag{2.3}$$

and from the metric (2.1) the zehnbein is chosen to be

$$e^{0,1,2,3} = u dx^{0,1,2,3}\,, \quad e^4 = \frac{du}{u}\,, \quad e^{a'} = \prod_{\alpha=1}^{a'-5} \sin\phi^\alpha d\phi^{a'-4}\,. \tag{2.4}$$

In addition to the metric (2.1), another constituent of the $\mathrm{AdS}_5\times\mathrm{S}^5$ background is the Ramond-Ramond five form field strength given by

$$F_5 = 4e^0 \wedge e^1 \wedge e^2 \wedge e^3 \wedge e^4 + 4e^5 \wedge e^6 \wedge e^7 \wedge e^8 \wedge e^9\,. \tag{2.5}$$

The $\mathrm{AdS}_5\times\mathrm{S}^5$ background composed of (2.1) and (2.5) is maximally supersymmetric. Its supersymmetry structure is encoded in the spacetime Killing spinor η^I, which is the solution of the spacetime Killing spinor equation $D_\mu\eta^I(X) = 0$ for the $\mathrm{AdS}_5\times\mathrm{S}^5$ background.[5] The Killing spinor equation has been solved in refs. [22, 23], and its solution is expressed in a simpler form if we split η^I as

$$\eta^I = \eta^I_+ + \eta^I_-\,, \tag{2.6}$$

where $\eta^I_\pm$ are defined by

$$\eta^I_\pm = P^{IJ}_\pm \eta^J \tag{2.7}$$

with the projection operator

$$P^{IJ}_\pm = \frac{1}{2}\left(\delta^{IJ} \pm \Gamma_{0123}\tau_2^{IJ}\right)\,. \tag{2.8}$$

In this splitting, we see that $\eta^1_\pm$ and $\eta^2_\pm$ are not independent from each other because

$$\eta^2_\pm = \mp\Gamma_{0123}\eta^1_\pm\,. \tag{2.9}$$

[5] The explicit form of the covariant derivative D_μ can be found in eq. (A.5).

Thus, to avoid this redundancy, it is convenient to define

$$\eta_\pm \equiv \eta^1_\pm \,, \tag{2.10}$$

to which η^1 and η^2 are related by

$$\eta^1 = \eta_+ + \eta_- \,, \quad \eta^2 = -\Gamma_{0123}\left(\eta_+ - \eta_-\right) \,, \tag{2.11}$$

as checked from eqs. (2.6), (2.9) and (2.10). If we now use $\eta_\pm$, then the solution of the Killing spinor equation is expressed as[6]

$$\begin{aligned}\eta_+(X) &= u^{1/2} S(\phi)\left(\epsilon_+ - x\cdot\Gamma\epsilon_-\right) \,, \\ \eta_-(X) &= u^{-1/2}\Gamma_4 S(\phi)\epsilon_- \,,\end{aligned} \tag{2.12}$$

where $\epsilon_\pm$ are constant spinors, $x\cdot\Gamma = x^0\Gamma_0 + x^1\Gamma_1 + x^2\Gamma_2 + x^3\Gamma_3$ (or $x^0\Gamma_0 + \vec{x}\cdot\vec{\Gamma}$), and $S(\phi)$ is a spinorial function of five angles of S^5 given by

$$S(\phi) = \prod_{a'=5}^{9} \exp\left(\frac{1}{2}\phi^{a'-4}\Gamma_{(a'-1)a'}\right) \,. \tag{2.13}$$

We note that, since η^I is taken to have positive chirality in this paper, $\epsilon_+(\epsilon_-)$ is a positive (negative) chirality spinor,

$$\Gamma^{11}\epsilon_\pm = \pm\epsilon_\pm \,, \tag{2.14}$$

and has sixteen independent free components.[7]

2.2 Symmetries of Dp-brane action

The Dp-brane action S_p is composed of the Dirac-Born-Infeld (DBI) and the Wess-Zumino (WZ) parts:

$$S_p = S_{\mathrm{DBI}} + S_{\mathrm{WZ}} \,, \tag{2.15}$$

where

$$S_{\mathrm{DBI}} = -\int_{\mathcal{M}_{p+1}} d^{p+1}\sigma\sqrt{-\det\left(G_{ij}+\mathcal{F}_{ij}\right)} \,, \quad S_{\mathrm{WZ}} = \int_{\mathcal{M}_{p+2}} H_{p+2} \,, \tag{2.16}$$

Here, $\mathcal{M}_{p+1}$ represents the Dp-brane worldvolume and $\mathcal{M}_{p+2}$ is a $(p+2)$-dimensional manifold whose boundary is identified with $\mathcal{M}_{p+1}$, that is, $\partial\mathcal{M}_{p+2} = \mathcal{M}_{p+1}$.

In the DBI part, G_{ij} is the pullback of the $\mathrm{AdS}_5\times\mathrm{S}^5$ supergeometry described by the Cartan one-form vector superfield $L^{\hat{a}}$ onto the worldvolume,

$$G_{ij} = L^{\hat{a}}_i L^{\hat{b}}_j \eta_{\hat{a}\hat{b}} \,, \quad L^{\hat{a}}_i = \partial_i Z^M L^{\hat{a}}_M \,, \tag{2.17}$$

where i,j are the worldvolume indices $(i,j = 0,1,\ldots,p)$. $\mathcal{F}_{ij}$ is a combination of the field strength F_{ij} of the worldvolume gauge field A_i $(F_{ij} = \partial_i A_j - \partial_j A_i)$ and the pulled-back background NS-NS two-form superfield $\mathcal{B}$. In the form notation, $\mathcal{F}$ is given by

$$\mathcal{F} = F - \mathcal{B} = dA + 2i\int_0^1 ds L^{\hat{a}}_s \wedge \bar{\Theta}^I\Gamma_{\hat{a}}\tau_3^{IJ} L^J_s \,, \tag{2.18}$$

[6] While the complex spinor notation is adopted in [22], we use the real or Majorana-Weyl spinor notation throughout the paper.

[7] The definition of Γ^{11} is given in (A.6).

where L^I is the Cartan one-form spinorial superfield and the subscript s in the superfields means that the fermionic coordinate Θ inside the superfields is replaced by $\Theta \to s\Theta$. In the WZ part, H_{p+2} is the supersymmetric closed $(p+2)$-form consisting of various combinations of the Cartan one-form superfields and $\mathcal{F}$.[8]

The Dp-brane action has three manifest symmetries. Firstly, it is invariant under the worldvolume reparametrization

$$\sigma^i \to \sigma^i - \lambda^i(\sigma)\,, \tag{2.19}$$

where $\lambda^i(\sigma)$ is the local reparametrization parameter. Under this, the worldvolume fields transform as follows.

$$\delta_\lambda \Theta^I = \lambda^i \partial_i \Theta^I\,, \quad \delta_\lambda X^\mu = \lambda^i \partial_i X^\mu\,, \quad \delta_\lambda A_i = \lambda^j \partial_j A_i + \partial_i \lambda^j A_j\,. \tag{2.20}$$

We note that both of Θ^I and X^μ are scalars from the worldvolume viewpoint. Secondly, the action is spacetime supersymmetric under the transformations

$$\delta_\eta Z^M L_M^{\hat{a}} = 2i\bar{\eta}^I \Gamma^{\hat{a}} \Theta^I\,, \quad \delta_\eta Z^M L_M^I = \eta^I\,, \tag{2.21}$$

where η^I is the Killing spinor of eq. (2.11) with eq. (2.12). More precisely, the DBI and the WZ parts of the action are supersymmetric separately. Actually, supersymmetry is natural because the super coset method respects the background supersymmetry by construction. If we expand the spacetime supersymmetry transformation of eq. (2.21) in terms of Θ, we get

$$\begin{aligned}
\delta_\eta \Theta^I &= \eta^I + \mathcal{O}(\Theta^2)\,, \\
\delta_\eta X^\mu &= -ie_{\hat{a}}^\mu \bar{\Theta}^I \Gamma^{\hat{a}} \eta^I + \mathcal{O}(\Theta^3)\,, \\
\delta_\eta A_i &= -ie_i^{\hat{a}} \bar{\Theta}^I \Gamma_{\hat{a}} \tau_3^{IJ} \eta^J + \mathcal{O}(\Theta^3)\,,
\end{aligned} \tag{2.22}$$

where $e_{\hat{a}}^\mu$ is the inverse of the zehnbein $e_\mu^{\hat{a}}$ given in eq. (2.4) and

$$e_i^{\hat{a}} = \partial_i X^\mu e_\mu^{\hat{a}}\,. \tag{2.23}$$

The transformation for the worldvolume gauge field A_i is determined from the invariance of $\mathcal{F}$ of eq. (2.18), $\delta_\eta \mathcal{F} = 0$ [6, 7].

The last one is the local fermionic κ symmetry, which is in some sense the most important one since it guarantees the worldvolume supersymmetry after gauge fixing. The κ symmetry transformation rules are given by

$$\delta_\kappa Z^M L_M^{\hat{a}} = 0\,, \quad \delta_\kappa Z^M L_M^I = \kappa^I\,, \tag{2.24}$$

where the transformation parameter κ satisfies, for the κ symmetric projection $\Gamma^{(p)}$,

$$\Gamma^{(p)IJ} \kappa^J = \kappa^I\,. \tag{2.25}$$

[8]For a comprehensive study on the WZ part, see for example ref. [24] where the systematic Chevalley-Eilenberg cohomology has been used to construct H_{p+2}.

JHEP09(2017)022

The κ symmetry projection is basically the pullback of various gamma matrix products onto the Dp-brane worldvolume and, for the type IIB case, its explicit expression [25] is

$$\Gamma^{(p)} = \frac{1}{\sqrt{-\det\left(G_{ij}+\mathcal{F}_{ij}\right)}} \sum_{n=0}^{(p+1)/2} \frac{1}{2^n n!} \gamma^{j_1 k_1 \cdots j_n k_n} \mathcal{F}_{j_1 k_1} \cdots \mathcal{F}_{j_n k_n} J^{(n)}_{(p)} ,$$
$$J^{(n)}_{(p)} = \frac{(-1)^n}{(p+1)!} \epsilon^{i_0 i_1 \cdots i_p} \gamma_{i_0 i_1 \cdots i_p} \tau_3^{n+(p-3)/2} \tau_2 , \tag{2.26}$$

where $\gamma_{i_1 \cdots i_n} = \gamma_{[i_1} \cdots \gamma_{i_n]}$ and γ_i is the pullback of $\Gamma_{\hat{a}}$, $\gamma_i = L_i^{\hat{a}} \Gamma_{\hat{a}}$. The important properties of $\Gamma^{(p)}$ are

$$\Gamma^{(p)2} = 1 , \quad \mathrm{Tr}\Gamma^{(p)} = 0 , \tag{2.27}$$

and, as is verified with the τ matrices (A.3), $\Gamma^{(p)}$ can always be put into the form

$$\Gamma^{(p)} = \begin{pmatrix} 0 & \beta_+^{(p)} \\ \beta_-^{(p)} & 0 \end{pmatrix} . \tag{2.28}$$

The two blocks $\beta_+^{(p)}$ and $\beta_-^{(p)}$ satisfy $\beta_+^{(p)} \beta_-^{(p)} = \beta_-^{(p)} \beta_+^{(p)} = 1$, which is $\Gamma^{(p)2} = 1$ of (2.27), and their expressions for each p will be given in the next section. If we now write down the κ symmetry transformation rules for the worldvolume fields by expanding eq. (2.24) in terms of Θ, then they are

$$\begin{aligned} \delta_\kappa \Theta^I &= \kappa^I + \mathcal{O}\left(\Theta^2\right) , \\ \delta_\kappa X^\mu &= i e^\mu_{\hat{a}} \bar{\Theta}^I \Gamma^{\hat{a}} \delta_\kappa \Theta^I + \mathcal{O}\left(\Theta^3\right) , \\ \delta_\kappa A_i &= i e_i^{\hat{a}} \bar{\Theta}^I \Gamma_{\hat{a}} \tau_3^{IJ} \delta_\kappa \Theta^J + \mathcal{O}\left(\Theta^3\right) , \end{aligned} \tag{2.29}$$

where the transformation rule for A_i is determined from $\delta_\kappa \mathcal{F} = -2i L^{\hat{a}} \wedge \bar{L}^I \Gamma_{\hat{a}} \tau_3^{IJ} \delta_\kappa \Theta^J$ [6, 7].[9]

2.3 Worldvolume supersymmetry

If a given configuration or embedding of Dp-brane in a specific supersymmetric background preserves some fraction of the background supersymmetries, then the preserved supersymmetries should be respected on the D-brane worldvolume theory. How are they realized and described? One way to answer this practical question is to follow the procedure developed in refs. [6, 7]. In this subsection, following refs. [6, 7], we describe how to identify the supersymmetry on the worldvolume and give the associated supersymmetry transformation rules for the worldvolume fields.

For a given Dp brane, we first consider its configuration based on the data of table 1 and align the worldvolume coordinates with those of spacetime as

$$X^{\ell_i}(\sigma) = \sigma^i \quad (i = 0, 1, \ldots, p) , \tag{2.30}$$

which is equivalent to specify indices $(\ell_0, \ell_1, \ldots \ell_p)$ among ten spacetime coordinates (2.3). This is nothing but the static gauge which fixes the worldvolume reparametrization symmetry (2.19). Since the Lorentzian branes are of our concern, X^{ℓ_0} will be always $X^0 (= x^0)$,

[9]The expression for $\delta_\kappa \mathcal{F}$ itself has bee derived in ref. [26].

that is, $\ell_0 = 0$ or $x^0(\sigma) = \sigma^0$. The remaining spacetime coordinates transverse to X^{ℓ_i} will be denoted by X^f, which describe the transverse fluctuations of Dp-brane. Since the brane may be placed in some transverse position, it is convenient to split X^f as

$$X^f = X_0^f + \tilde{X}^f , \tag{2.31}$$

where X_0^f denote the constant transverse position of brane and $\tilde{X}^f$ are the fluctuations around them. We note that we could consider more general configurations where X^f depend on X^{ℓ_i} as $X^f = X^f(X^{\ell_i})$. One typical example would be the constant motion along certain transverse directions: $\partial_0 X^f =$ constant. In this paper, however, we will restrict ourselves to purely static configurations and turn off any worldvolume fluxes

As alluded to in the last subsection, the Dp brane has a local worldvolume symmetry, the κ symmetry. As we do in a theory with local gauge symmetries, we should fix it properly before doing any actual calculation. Here, we take the covariant κ symmetry fixing condition given by

$$\Theta^1 = 0 , \quad \Theta^2 = \theta , \tag{2.32}$$

or $(1 + \tau_3)^{IJ}\Theta^J = 0$. This condition is an admissible one because it is in accord with the criterion for the admissible fixing condition [25]: τ_3 does not commute with $\Gamma^{(p)}$ of the κ symmetry projector (2.25), $[\tau_3, \Gamma^{(p)}] \neq 0$. Though the covariant fixing condition is not so helpful in simplifying the D-brane action, it is convenient to explore the symmetry structure in a covariant way.[10]

Having fixed the reparametrization and the κ symmetries, we look at the spacetime supersymmetry transformation (2.22). Then we easily see that the transformation violates the gauge-fixing conditions of eqs. (2.30) and (2.32) because $\delta_\eta \Theta^1 \neq 0$ and $\delta_\eta X^{\ell_i} \neq 0$. One possible way of resolving this situation is to introduce the compensating κ and the worldvolume reparametrization transformations and define a new transformation δ as

$$\delta \equiv \delta_\eta + \delta_\kappa + \delta_\lambda . \tag{2.33}$$

The parameters κ and λ of the compensating transformations are determined in terms of η^I such that the new transformation δ keeps the gauge-fixing conditions, that is, $\delta\Theta^1 = 0$ and $\delta X^{\ell_i} = 0$. They can be found order by oder in θ and are, at the leading order,

$$\begin{aligned} &\kappa^1 = -\eta^1 + \mathcal{O}\left(\theta^2\right) , \qquad \kappa^2 = -\beta_-^{(p)}\eta^1 + \mathcal{O}\left(\theta^2\right) , \\ &\lambda^i = i e_{\hat{a}}^{\ell_i} \bar{\theta}\Gamma^{\hat{a}} \left(\eta^2 + \beta_-^{(p)}\eta^1\right) + \mathcal{O}\left(\theta^3\right) , \end{aligned} \tag{2.34}$$

where $\beta_-^{(p)}$ appears due to eq. (2.28). In this way, we have the transformation δ consistent with the gauge-fixing conditions and interpret it as the worldvolume supersymmetry.

Let us now turn to the theory on the Dp-brane worldvolume. From the viewpoint of the worldvolume theory, the static gauge describing the embedding of the brane can be regarded as the 'vacuum' configuration. Then, as usual, the supersymmetry preserved by

[10]See for example ref. [27], where the covariant gauge is adopted in studying the D3-brane in the plane wave background.

the 'vacuum' is specified by the free components of the supersymmetry parameter satisfying the equation $\delta(\text{fermion}) = 0$, that is, $\delta\theta = 0$, which we call the worldvolume Killing spinor equation. If we rewrite $\delta\theta = 0$ by plugging (2.20), (2.22), (2.29) into (2.33) with (2.34), it becomes a fairly simple equation,

$$0 = \eta^2 - \beta_0^{(p)} \eta^1 \,, \tag{2.35}$$

where $\beta_0^{(p)}$ depends on the 'vacuum' configuration and is defined by

$$\beta_0^{(p)} \equiv \beta_-^{(p)} \Big|_{\theta=0,\ A_i=0,\ X^f=X_0^f} \,. \tag{2.36}$$

We would like to note that the worldvolume Killing spinor equation (2.35) is an exact one because $\theta = 0$ in the 'vacuum' configuration.

Further evaluation of (2.35) using (2.11) and (2.12) leads us to have

$$\begin{aligned} 0 =& -\left(\Gamma_{0123} + \beta_0^{(p)}\right)\eta_+ + \left(\Gamma_{0123} - \beta_0^{(p)}\right)\eta_- \\ =& -u^{1/2}\Gamma_{0123}\left(1 - \Gamma_{0123}\beta_0^{(p)}\right) S_0(\phi)\left(\epsilon_+ - x\cdot\Gamma\epsilon_-\right) \\ &+ u^{-1/2}\Gamma_{0123}\left(1 + \Gamma_{0123}\beta_0^{(p)}\right)\Gamma_4 S_0(\phi)\epsilon_- \,, \end{aligned} \tag{2.37}$$

where some coordinates among u and $x^{1,2,3}$ are understood to be constants if they are transverse directions, and $S_0(\phi)$ means $S(\phi)$ of (2.13) in which the angular directions included in X^f are set to constant values. In the investigation of the worldvolume supersymmetry structure of a 'vacuum' configuration, the first step is to check if the square of $\Gamma_{0123}\beta_0^{(p)}$ in eq. (2.37) is equal to one, $\left(\Gamma_{0123}\beta_0^{(p)}\right)^2 = 1$. If it is the case, $\Gamma_{0123}\beta_0^{(p)}$ has eigenvalues of ± 1. This means that $1 \pm \Gamma_{0123}\beta_0^{(p)}$ play the role of projection operators and give us the possibility of identifying which components of $\epsilon_\pm$ are free. The next step is to send $\Gamma_{0123}\beta_0^{(p)}$ to the right of $S_0(\phi)$, which is done by evaluating $S_0^{-1}(\phi)\Gamma_{0123}\beta_0^{(p)} S_0(\phi)$ with repeated use of the following identity

$$e^{-\frac{1}{2}\phi\Gamma_{a(a+1)}}\Gamma_a e^{\frac{1}{2}\phi\Gamma_{a(a+1)}} = \Gamma_a \cos\phi + \Gamma_{a+1}\sin\phi \,. \tag{2.38}$$

Generically, the resulting expression is not of the form of projection operator but a sum of many gamma matrix products with coefficients composed of trigonometric functions. However, as we will see in the next section, it becomes a projection operator for some special values of the transverse angular coordinates and can be used to pick out the free components among $\epsilon_\pm$. This is interesting in a sense that the transverse position is determined by insisting on the supersymmetry in the D-brane worldvolume theory without resort to the equations of motion.[11]

After identifying the supersymmetry preserved on the D-brane worldvolume, we can read off how the transformation δ acts on the remaining worldvolume fields. Let us denote the worldvolume fields collectively as

$$\Phi = \left(\tilde{X}^f, A_i, \theta\right) \,. \tag{2.39}$$

[11]Strictly speaking, in some cases, we should also impose the non-degeneracy condition for the induced worldvolume metric.

Its transformation $\delta\Phi$ can be written as an expansion in terms of the power of Φ. If we consider the terms up to linear order in Φ, then the transformation rules for the worldvolume fields are

$$\begin{aligned}
\delta\tilde{X}^f &= -ie^f_{\hat{a}}\bar{\theta}\Gamma^{\hat{a}}\left(\hat{\eta}^2 + \beta^{(p)}_-\hat{\eta}^1\right) + \mathcal{O}(\Phi^2)\ , \\
\delta A_i &= ie^{\hat{a}}_i\bar{\theta}\Gamma_{\hat{a}}\left(\hat{\eta}^2 + \beta^{(p)}_-\hat{\eta}^1\right) + \mathcal{O}(\Phi^2)\ , \\
\delta\theta &= \hat{\eta}^2 - \beta^{(p)}_-\hat{\eta}^1 + \mathcal{O}(\Phi^2)\ ,
\end{aligned} \tag{2.40}$$

where $\hat{\eta}^{1,2}$ are $\eta^{1,2}$ containing only the free components of $\epsilon_\pm$ picked out from the worldvolume Killing spinor equation (2.37) and we have omitted the compensating worldvolume reparametrization transformation because it begins with the terms quadratic order in Φ. In these transformation rules, $\beta^{(p)}_-$ is also expanded as

$$\beta^{(p)}_- = \beta^{(p)}_0 + \beta^{(p)}_1 + \mathcal{O}(\Phi^2)\ , \tag{2.41}$$

where $\beta^{(p)}_0$ is defined in (2.36) and $\beta^{(p)}_1$ is the collection of terms linear order in Φ.

In the process of calculation leading to (2.40), we will encounter many trigonometric functions. Thus, for notational simplicity, we would like to define the following quantities before moving on to the next section.

$$\begin{aligned}
s_\alpha &\equiv \sin\phi^\alpha\ , \qquad & c_\alpha &\equiv \cos\phi^\alpha\ , \\
\mathring{s}_\alpha &\equiv \sin\phi^\alpha_0\ , \qquad & \mathring{c}_\alpha &\equiv \cos\phi^\alpha_0\ ,
\end{aligned} \tag{2.42}$$

3 AdS branes

When the radial direction of the AdS space u is one of the worldvolume directions for a given brane configuration, the brane is usually called the AdS brane since the induced metric on the worldvolume contains the AdS space. If we take a look at the table 1 and consider the Lorentzian branes in which the time x^0 is always a worldvolume direction, we see that there are six types of AdS brane configurations. In this section, for each of them, following the procedure outlined in section 2.3, we investigate the supersymmetry realized on the worldvolume and give the worldvolume supersymmetry transformation rules for the worldvolume fields. We note that the following subsections and subsubsections are self-contained and completely independent from each other.

3.1 D1

The D1-brane configuration (2,0) of table 1 leads us to take the static gauge as

$$x^0(\sigma) = \sigma^0\ , \quad u(\sigma) = \sigma^1\ , \tag{3.1}$$

or $(\ell_0, \ell_1) = (0, 4)$ in eq. (2.30), which corresponds to the AdS_2 brane. The coordinates transverse to this configuration are then

$$X^f = \{x^1, x^2, x^3, \phi^1, \dots, \phi^5\}\ . \tag{3.2}$$

In order to identify which part of the spacetime supersymmetry is preserved on the worldvolume of AdS_2 brane, we should solve the worldvolume Killing spinor equation (2.35). What is necessary to do this is $\beta^{(1)}_{\pm}$, which is read off from eqs. (2.26) and (2.28) as

$$\beta^{(1)}_{\pm} = \frac{\epsilon^{i_1 i_2}}{2\sqrt{-\det(G_{ij}+\mathcal{F}_{ij})}}\Big(\gamma_{i_1 i_2} \pm \mathcal{F}_{i_1 i_2}\Big)\,. \tag{3.3}$$

According to (2.36), we then see that this expression leads to

$$\beta^{(1)}_{0} = -\Gamma_{04} \tag{3.4}$$

in the static gauge (3.1). Now $\left(\Gamma_{0123}\beta^{(1)}_0\right)^2 = 1$ obviously and thus $1 \pm \Gamma_{0123}\beta^{(1)}_0$ in the worldvolume Killing spinor equation (2.37) with $p = 1$ play the role of projection operators. Having the projection operators, the next step described in section 2.3 is to send $\Gamma_{0123}\beta^{(1)}_0$ to the right of $S_0(\phi)$ in (2.37). If we denote the resulting expression as $\tilde{\Gamma}$, we get the relation $\Gamma_{0123}\beta^{(1)}_0 S_0(\phi) = S_0(\phi)\tilde{\Gamma}$. With the fact that $\Gamma_{0123}\beta^{(1)}_0 = -\Gamma_{1234}$, $\tilde{\Gamma}$ is evaluated by repeated use of the identity (2.38) as follows:

$$\begin{aligned}
-\tilde{\Gamma} &= S_0^{-1}(\phi)\Gamma_{1234}S_0(\phi) \\
&= \mathring{c}_1\Gamma_{1234} + \mathring{s}_1\mathring{c}_2\Gamma_{1235} + \mathring{s}_1\mathring{s}_2\mathring{c}_3\Gamma_{1236} + \mathring{s}_1\mathring{s}_2\mathring{s}_3\mathring{c}_4\Gamma_{1237} \\
&\quad + \mathring{s}_1\mathring{s}_2\mathring{s}_3\mathring{s}_4\mathring{c}_5\Gamma_{1238} + \mathring{s}_1\mathring{s}_2\mathring{s}_3\mathring{s}_4\mathring{s}_5\Gamma_{1239}\,,
\end{aligned} \tag{3.5}$$

where we have used the definitions of eq. (2.42). However, this shows clearly that $1 \pm \tilde{\Gamma}$ do not have the form of projection operators. At this stage, we are required to fix the transverse angular position properly in such a way that makes them have the desired form. Although there are various possibilities, we fix the angular position to be $\phi^\alpha_0 = \frac{\pi}{2}$ ($\alpha = 1, \ldots, 5$) for simplicity, since all the points on S^5 are equivalent and the AdS_2 brane is a point on S^5. For this angular position, $\tilde{\Gamma} = -\Gamma_{1239}$. If we now split $\epsilon_{\pm}$ according to the eigenvalues of Γ_{1234} as

$$\Gamma_{1239}\epsilon_{+\pm} = \pm\epsilon_{+\pm}\,, \quad \Gamma_{1239}\epsilon_{-\pm} = \pm\epsilon_{-\pm}\,, \tag{3.6}$$

then the worldvolume Killing spinor equation (2.37) becomes

$$0 = -2u^{1/2}\Gamma_{0123}S_0(\phi)\left(\epsilon_{++} - x^0\Gamma_0\epsilon_{-+} - \vec{x}_0\cdot\vec{\Gamma}\epsilon_{--}\right) + 2u^{-1/2}\Gamma_{0123}\Gamma_4 S_0(\phi)\epsilon_{-+}\,, \tag{3.7}$$

with

$$S_0(\phi) = S(\phi)|_{\phi^1,\ldots,\phi^5=\pi/2}\,. \tag{3.8}$$

The solution of this equation is readily found to be

$$\epsilon_{-+} = 0\,, \quad \epsilon_{++} = \vec{x}_0\cdot\vec{\Gamma}\epsilon_{--}\,. \tag{3.9}$$

Since other components except for those of (3.9) are undetermined, we conclude that the supersymmetry preserved on the AdS_2 brane is characterized by

$$\epsilon_{+-}\,, \quad \epsilon_{--}\,, \tag{3.10}$$

JHEP09(2017)022

each of which has eight free components and we have sixteen supersymmetries (1/2-BPS) in total.

Having identified the worldvolume supersymmetries (3.10), it is straightforward to obtain the supersymmetry transformation rules for the worldvolume fields according to (2.40) and (2.41). Firstly, the scalar fields corresponding to the transverse fluctuations are found to transform as

$$\begin{aligned}\delta\tilde{\vec{x}} &= 2iu^{-1}\bar{\theta}\vec{\Gamma}\Gamma_{0123}\left(\hat{\eta}_{+}-\hat{\eta}_{-}\right)+\dots\,,\\ \delta\tilde{\phi}^{\alpha} &= 2i\bar{\theta}\Gamma_{\alpha+4}\Gamma_{0123}\left(\hat{\eta}_{+}-\hat{\eta}_{-}\right)+\dots\,,\end{aligned}\tag{3.11}$$

where $\alpha = 1,\dots,5$ and

$$\begin{aligned}\hat{\eta}_{+} &= u^{1/2}S_0(\phi)\left(\epsilon_{+-}-x^0\Gamma_0\epsilon_{--}\right)\,,\\ \hat{\eta}_{-} &= u^{-1/2}\Gamma_4 S_0(\phi)\epsilon_{--}\,.\end{aligned}\tag{3.12}$$

As for the worldvolume gauge field, the transformation rule is obtained as

$$\begin{aligned}\delta A_0 &= -2iu\bar{\theta}\Gamma_0\Gamma_{0123}\left(\hat{\eta}_{+}-\hat{\eta}_{-}\right)+\dots\,,\\ \delta A_u &= -2iu^{-1}\bar{\theta}\Gamma_4\Gamma_{0123}\left(\hat{\eta}_{+}-\hat{\eta}_{-}\right)+\dots\,.\end{aligned}\tag{3.13}$$

Finally, we get the transformation rule for the fermionic field as

$$\delta\theta = -2u\tilde{\vec{x}}\cdot\vec{\Gamma}\Gamma_{01234}\hat{\eta}_{-}+\tilde{\phi}^{\alpha}\Gamma_{\alpha+4}\Gamma_{01234}\left(\hat{\eta}_{+}+\hat{\eta}_{-}\right)-\beta_1^{(1)}\left(\hat{\eta}_{+}+\hat{\eta}_{-}\right)+\dots\,,\tag{3.14}$$

where $\alpha = 1,\dots,5$ and

$$\beta_1^{(1)} = \left(\frac{1}{u}\Gamma^0\partial_0\tilde{X}^f+u\Gamma^4\partial_u\tilde{X}^f\right)e_f^{\hat{a}}\Gamma_{04\hat{a}}+F_{0u}\,.\tag{3.15}$$

3.2 D3

The D3-brane configuration (3,1) of table 1 leads us to take the static gauge as

$$x^{0,1}(\sigma)=\sigma^{0,1}\,,\quad u(\sigma)=\sigma^2\,,\quad \phi^5(\sigma)=\sigma^3\,,\tag{3.16}$$

or $(\ell_0,\ell_1,\ell_2,\ell_3)=(0,1,4,9)$ in eq. (2.30), which corresponds to the $AdS_3\times S^1$ brane. The coordinates transverse to this configuration are then

$$X^f = \left\{x^2,x^3,\phi^1,\dots,\phi^4\right\}\,.\tag{3.17}$$

In order to identify which part of the spacetime supersymmetry is preserved on the worldvolume of $AdS_3\times S^1$ brane, we should solve the worldvolume Killing spinor equation (2.35). What is necessary to do this is $\beta_\pm^{(3)}$, which is read off from eqs. (2.26) and (2.28) as

$$\beta_\pm^{(3)} = \frac{\epsilon^{i_1\cdots i_4}}{\sqrt{-\det\left(G_{ij}+\mathcal{F}_{ij}\right)}}\left(\pm\frac{1}{4!}\gamma_{i_1\cdots i_4}+\frac{1}{4}\gamma_{i_1i_2}\mathcal{F}_{i_3i_4}\pm\frac{1}{8}\mathcal{F}_{i_1i_2}\mathcal{F}_{i_3i_4}\right).\tag{3.18}$$

JHEP09(2017)022

According to (2.36), we then see that this expression leads to

$$\beta_0^{(3)} = \Gamma_{0149} \tag{3.19}$$

in that static gauge (3.16). Now $\left(\Gamma_{0123}\beta_0^{(3)}\right)^2 = 1$ obviously and thus $1 \pm \Gamma_{0123}\beta_0^{(3)}$ in the worldvolume Killing spinor equation (2.37) with $p = 3$ play the role of projection operators. Having the projection operators, the next step described in section 2.3 is to send $\Gamma_{0123}\beta_0^{(3)}$ to the right of $S_0(\phi)$ in (2.37). If we denote the resulting expression as $\tilde{\Gamma}$, we get the relation $\Gamma_{0123}\beta_0^{(3)} S_0(\phi) = S_0(\phi)\tilde{\Gamma}$. With the fact that $\Gamma_{0123}\beta_0^{(3)} = \Gamma_{2349}$, $\tilde{\Gamma}$ is evaluated by repeated use of the identity (2.38) as follows:

$$\begin{aligned}
\tilde{\Gamma} &= S_0^{-1}(\phi)\Gamma_{2349}S_0(\phi) \\
&= \mathring{c}_1\Gamma_{2349} + \mathring{s}_1\mathring{c}_2\Gamma_{2359} + \mathring{s}_1\mathring{s}_2\mathring{c}_3\Gamma_{2369} - \mathring{s}_1\mathring{s}_2\mathring{s}_3\mathring{c}_4 s_5\Gamma_{2378} \\
&\quad + \mathring{s}_1\mathring{s}_2\mathring{s}_3\mathring{c}_4 c_5\Gamma_{2379} + \mathring{s}_1\mathring{s}_2\mathring{s}_3\mathring{s}_4\Gamma_{2389}\,,
\end{aligned} \tag{3.20}$$

where we have used the definitions of eq. (2.42). However, this shows clearly that $1 \pm \tilde{\Gamma}$ do not have the form of projection operators. One can make them have the desired form by fixing the transverse angular position, and realize that there are four possible choices which are (i) $\phi_0^1 = 0$, $\phi_0^{2,3,4}$ =arbitrary, (ii) $\phi_0^1 = \frac{\pi}{2}$, $\phi_0^2 = 0$, $\phi_0^{3,4}$ =arbitrary, (iii) $\phi_0^{1,2} = \frac{\pi}{2}$, $\phi_0^3 = 0$, ϕ_0^4 =arbitrary, (iv) $\phi_0^{1,2,3,4} = \frac{\pi}{2}$. Except for the last one, the first three choices lead to the singular or degenerate induced worldvolume metric. Thus, if one wishes to have a regular theory on the worldvolume, the last choice is quite natural and hence $\tilde{\Gamma} = \Gamma_{2389}$. If we now split $\epsilon_\pm$ according to the eigenvalues of Γ_{2389} as

$$\Gamma_{2389}\epsilon_{+\pm} = \pm\epsilon_{+\pm}\,, \quad \Gamma_{2389}\epsilon_{-\pm} = \pm\epsilon_{-\pm}\,, \tag{3.21}$$

then the worldvolume Killing spinor equation (2.37) becomes

$$\begin{aligned}
0 = &- 2u^{1/2}\Gamma_{0123}S_0(\phi)\left[\epsilon_{+-} - \left(x^0\Gamma_0 + x^1\Gamma_1\right)\epsilon_{--} - \left(x_0^2\Gamma_2 + x_0^3\Gamma_3\right)\epsilon_{-+}\right] \\
&+ 2u^{-1/2}\Gamma_{0123}\Gamma_4 S_0(\phi)\epsilon_{--}\,,
\end{aligned} \tag{3.22}$$

with

$$S_0(\phi) = S(\phi)|_{\phi^1,\ldots,\phi^4=\pi/2}\,. \tag{3.23}$$

The solution of this equation is readily found to be

$$\epsilon_{--} = 0\,, \quad \epsilon_{+-} = \left(x_0^2\Gamma_2 + x_0^3\Gamma_3\right)\epsilon_{-+}\,. \tag{3.24}$$

Since other components except for those of (3.24) are undetermined, we conclude that the supersymmetry preserved on the $\mathrm{AdS}_3{\times}\mathrm{S}^1$ brane is characterized by

$$\epsilon_{++}\,, \quad \epsilon_{-+}\,, \tag{3.25}$$

each of which has eight free components and we have sixteen supersymmetries (1/2-BPS) in total.

Having identified the worldvolume supersymmetries (3.25), it is straightforward to obtain the supersymmetry transformation rules for the worldvolume fields according to (2.40)

JHEP09(2017)022

and (2.41). Firstly, the scalar fields corresponding to the transverse fluctuations are found to transform as

$$\begin{aligned}\delta\tilde{x}^{2,3} &= 2iu^{-1}\bar{\theta}\Gamma^{2,3}\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \dots\,,\\ \delta\tilde{\phi}^{1,2,3,4} &= 2i\bar{\theta}\Gamma^{5,6,7,8}\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \dots\,,\end{aligned} \tag{3.26}$$

where

$$\begin{aligned}\hat{\eta}_+ &= u^{1/2}S_0(\phi)\left[\epsilon_{++} - \left(x^0\Gamma_0 + x^1\Gamma_1\right)\epsilon_{-+}\right]\,,\\ \hat{\eta}_- &= u^{-1/2}\Gamma_4 S_0(\phi)\epsilon_{-+}\,.\end{aligned} \tag{3.27}$$

As for the worldvolume gauge field, the transformation rule is obtained as

$$\begin{aligned}\delta A_{0,1} &= -2iu\bar{\theta}\Gamma_{0,1}\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \dots\,,\\ \delta A_u &= -2iu^{-1}\bar{\theta}\Gamma_4\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \dots\,,\\ \delta A_{\phi^5} &= -2i\bar{\theta}\Gamma_9\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \dots\,,\end{aligned} \tag{3.28}$$

Finally, we get the transformation rule for the fermionic field as

$$\delta\theta = -2u\left(\tilde{x}^2\Gamma_2 + \tilde{x}^3\Gamma_3\right)\Gamma_{01234}\hat{\eta}_- + \tilde{\phi}^\alpha\Gamma_{\alpha+4}\Gamma_{01234}\left(\hat{\eta}_+ + \hat{\eta}_-\right) - \beta_1^{(3)}\left(\hat{\eta}_+ + \hat{\eta}_-\right) + \dots\,, \tag{3.29}$$

where $\alpha = 1, 2, 3, 4$ and

$$\begin{aligned}\beta_1^{(3)} = &-\left(\frac{1}{u}\Gamma^0\partial_0\tilde{X}^f + \frac{1}{u}\Gamma^1\partial_1\tilde{X}^f + u\Gamma^4\partial_u\tilde{X}^f + \Gamma^9\partial_{\phi^5}\tilde{X}^f\right)e_f^{\hat{a}}\Gamma_{0149\hat{a}}\\ &- u\Gamma_{01}F_{u\phi^5} + \frac{1}{u}\Gamma_{04}F_{1\phi^5} - \Gamma_{09}F_{1u} - \frac{1}{u}\Gamma_{14}F_{0\phi^5} + \Gamma_{19}F_{0u} - \frac{1}{u^2}\Gamma_{49}F_{01}\,.\end{aligned} \tag{3.30}$$

One may wonder if the D-brane configuration considered above is stable since S^1 in S^5 is not a topological cycle. The similar problem was worked out by [28]. The scalar mode corresponding to slipping off the S^1 in S^5 satisfies Breitenlohner-Friedmann bound, hence it does not lead to the instability. Also the above D-brane configuration satisfies the so called generalized calibration, which is the condition for the supersymmetric cycle of D-branes to satisfy on the general supergravity background with various fluxes. [9]. These remarks hold as well for other D-brane configurations in subsequent subsections.

3.3 D5

In this subsection, we are led to consider two kinds of D5-brane configurations. The common content for them is $\beta_\pm^{(5)}$ appearing in the κ symmetry projection $\Gamma^{(5)}$, which is read off from eqs. (2.26) and (2.28) as

$$\begin{aligned}\beta_\pm^{(5)} = \frac{\epsilon^{i_1\cdots i_6}}{\sqrt{-\det\left(G_{ij} + \mathcal{F}_{ij}\right)}}\bigg(&\frac{1}{6!}\gamma_{i_1\cdots i_6} \pm \frac{1}{48}\gamma_{i_1\cdots i_4}\mathcal{F}_{i_5 i_6} + \frac{1}{16}\gamma_{i_1 i_2}\mathcal{F}_{i_3 i_4}\mathcal{F}_{i_5 i_6}\\ &\pm \frac{1}{48}\mathcal{F}_{i_1 i_2}\mathcal{F}_{i_3 i_4}\mathcal{F}_{i_5 i_6}\bigg)\,.\end{aligned} \tag{3.31}$$

3.3.1 $(4,2)$-brane

The D5-brane configuration (4,2) of table 1 leads us to take the static gauge as

$$x^{0,1,2}(\sigma) = \sigma^{0,1,2}\,, \quad u(\sigma) = \sigma^3\,, \quad \phi^{4,5}(\sigma) = \sigma^{4,5}\,, \tag{3.32}$$

or $(\ell_0, \ell_1, \ell_2, \ell_3, \ell_4, \ell_5) = (0, 1, 2, 4, 8, 9)$ in eq. (2.30), which corresponds to the $\mathrm{AdS}_4\times\mathrm{S}^2$ brane. The coordinates transverse to this configuration are then

$$X^f = \left\{x^3, \phi^1, \phi^2, \phi^3\right\}\,. \tag{3.33}$$

In the static gauge (3.32), $\beta_-^{(5)}$ of (3.31) becomes,

$$\beta_0^{(5)} = -\Gamma_{012489}\,, \tag{3.34}$$

according to (2.36). Now $\left(\Gamma_{0123}\beta_0^{(5)}\right)^2 = 1$ obviously and thus $1\pm\Gamma_{0123}\beta_0^{(5)}$ in the worldvolume Killing spinor equation (2.37) with $p = 5$ play the role of projection operators. Having the projection operators, the next step described in section 2.3 is to send $\Gamma_{0123}\beta_0^{(5)}$ to the right of $S_0(\phi)$ in (2.37). If we denote the resulting expression as $\tilde{\Gamma}$, we get the relation $\Gamma_{0123}\beta_0^{(5)}S_0(\phi) = S_0(\phi)\tilde{\Gamma}$. With the fact that $\Gamma_{0123}\beta_0^{(5)} = \Gamma_{3489}$, $\tilde{\Gamma}$ is evaluated by repeated use of the identity (2.38) as follows:

$$\begin{aligned}\tilde{\Gamma} &= S_0^{-1}(\phi)\Gamma_{3489}S_0(\phi)\\ &= \mathring{c}_1c_4\Gamma_{3489} + \mathring{c}_1s_4s_5\Gamma_{3478} - \mathring{c}_1s_4c_5\Gamma_{3479}\\ &\quad + \mathring{s}_1\mathring{c}_2c_4\Gamma_{3589} + \mathring{s}_1\mathring{c}_2s_4s_5\Gamma_{3578} - \mathring{s}_1\mathring{c}_2s_4c_5\Gamma_{3579}\\ &\quad + \mathring{s}_1\mathring{s}_2\mathring{c}_3c_4\Gamma_{3689} + \mathring{s}_1\mathring{s}_2\mathring{c}_3s_4s_5\Gamma_{3678} - \mathring{s}_1\mathring{s}_2\mathring{c}_3s_4c_5\Gamma_{3679}\\ &\quad + \mathring{s}_1\mathring{s}_2\mathring{s}_3\Gamma_{3789}\,,\end{aligned} \tag{3.35}$$

where we have used the definitions of eq. (2.42). This shows clearly that $1\pm\tilde{\Gamma}$ do not have the form of projection operators. One can make them have the desired form by fixing the transverse angular position, and realize that there is a unique choice of $\phi_0^{1,2,3} = \frac{\pi}{2}$ which leads to $\tilde{\Gamma} = \Gamma_{3789}$. If we now split $\epsilon_\pm$ according to the eigenvalues of Γ_{3789} as

$$\Gamma_{3789}\epsilon_{+\pm} = \pm\epsilon_{+\pm}\,, \quad \Gamma_{3789}\epsilon_{-\pm} = \pm\epsilon_{-\pm}\,, \tag{3.36}$$

then the worldvolume Killing spinor equation (2.37) becomes

$$\begin{aligned}0 = &-2u^{1/2}\Gamma_{0123}S_0(\phi)\left[\epsilon_{+-} - \left(x^0\Gamma_0 + x^1\Gamma_1 + x^2\Gamma_2\right)\epsilon_{--} - x_0^3\Gamma_3\epsilon_{-+}\right]\\ &+ 2u^{-1/2}\Gamma_{0123}\Gamma_4S_0(\phi)\epsilon_{--}\,,\end{aligned} \tag{3.37}$$

with

$$S_0(\phi) = S(\phi)|_{\phi^1,\phi^2,\phi^3=\pi/2}\,. \tag{3.38}$$

The solution of this equation is readily found to be

$$\epsilon_{--} = 0\,, \quad \epsilon_{+-} = x_0^3\Gamma_3\epsilon_{-+}\,. \tag{3.39}$$

Since other components except for those of (3.39) are undetermined, we conclude that the supersymmetry preserved on the $\mathrm{AdS}_4\times\mathrm{S}^2$ brane is characterized by

$$\epsilon_{++}\,,\quad \epsilon_{-+}\,, \tag{3.40}$$

each of which has eight free components and we have sixteen supersymmetries (1/2-BPS) in total.

Having identified the worldvolume supersymmetries (3.40), it is straightforward to obtain the supersymmetry transformation rules for the worldvolume fields according to (2.40) and (2.41). Firstly, the scalar fields corresponding to the transverse fluctuations are found to transform as

$$\begin{aligned} \delta\tilde{x}^3 &= 2iu^{-1}\bar{\theta}\Gamma^3\Gamma_{0123}\left(\hat{\eta}_+-\hat{\eta}_-\right)+\dots\,,\\ \delta\tilde{\phi}^{1,2,3} &= 2i\bar{\theta}\Gamma^{5,6,7}\Gamma_{0123}\left(\hat{\eta}_+-\hat{\eta}_-\right)+\dots\,. \end{aligned} \tag{3.41}$$

where

$$\begin{aligned} \hat{\eta}_+ &= u^{1/2}S_0(\phi)\left[\epsilon_{++}-\left(x^0\Gamma_0+x^1\Gamma_1+x^2\Gamma_2\right)\epsilon_{-+}\right]\,,\\ \hat{\eta}_- &= u^{-1/2}\Gamma_4S_0(\phi)\epsilon_{-+}\,. \end{aligned} \tag{3.42}$$

As for the worldvolume gauge field, we obtain

$$\begin{aligned} \delta A_{0,1,2} &= -2iu\bar{\theta}\Gamma_{0,1,2}\Gamma_{0123}\left(\hat{\eta}_+-\hat{\eta}_-\right)+\dots\,,\\ \delta A_u &= -2iu^{-1}\bar{\theta}\Gamma_4\Gamma_{0123}\left(\hat{\eta}_+-\hat{\eta}_-\right)+\dots\,,\\ \delta A_{\phi^4} &= -2i\bar{\theta}\Gamma_8\Gamma_{0123}\left(\hat{\eta}_+-\hat{\eta}_-\right)+\dots\,.\\ \delta A_{\phi^5} &= -2is_4\bar{\theta}\Gamma_9\Gamma_{0123}\left(\hat{\eta}_+-\hat{\eta}_-\right)+\dots\,. \end{aligned} \tag{3.43}$$

Finally, we get the transformation rule for the fermionic field as

$$\delta\theta = -2u\tilde{x}^3\Gamma_3\Gamma_{01234}\hat{\eta}_- + \tilde{\phi}^\alpha\Gamma_{\alpha+4}\Gamma_{01234}\left(\hat{\eta}_++\hat{\eta}_-\right) - \beta_1^{(5)}\left(\hat{\eta}_++\hat{\eta}_-\right)+\dots \tag{3.44}$$

where $\alpha=1,2,3$ and

$$\begin{aligned} \beta_1^{(5)} = &\left(\frac{1}{u}\Gamma^0\partial_0\tilde{X}^f+\frac{1}{u}\Gamma^1\partial_1\tilde{X}^f+\frac{1}{u}\Gamma^2\partial_2\tilde{X}^f+u\Gamma^4\partial_u\tilde{X}^f\right.\\ &\left.+\Gamma^8\partial_{\phi^4}\tilde{X}^f+\frac{1}{s_4}\Gamma^9\partial_{\phi^5}\tilde{X}^f\right)e^{\hat{a}}_f\Gamma_{012489\hat{a}}\\ &-\frac{1}{48u^2s_4}\epsilon^{i_0\cdots i_3i_4i_5}\left(e^{\hat{a}_0}_{\ell_{i_0}}\cdots e^{\hat{a}_3}_{\ell_{i_3}}\right)\bigg|_{\phi^1,\phi^2,\phi^3=\pi/2}\Gamma_{\hat{a}_0\cdots\hat{a}_3}F_{i_4i_5}\,. \end{aligned} \tag{3.45}$$

3.3.2 $(2,4)$-brane

The D5-brane configuration (2,4) of table 1 leads us to take the static gauge as

$$x^0(\sigma)=\sigma^0\,,\quad u(\sigma)=\sigma^1\,,\quad \phi^{2,3,4,5}(\sigma)=\sigma^{2,3,4,5}\,, \tag{3.46}$$

or $(\ell_0, \ell_1, \ell_2, \ell_3, \ell_4, \ell_5) = (0, 4, 6, 7, 8, 9)$ in eq. (2.30), which corresponds to the $\mathrm{AdS}_2 \times \mathrm{S}^4$ brane. The coordinates transverse to this configuration are then

$$X^f = \{x^1, x^2, x^3, \phi^1\} \tag{3.47}$$

In the static gauge (3.46), $\beta_-^{(5)}$ of (3.31) becomes,

$$\beta_0^{(5)} = -\Gamma_{046789}\,, \tag{3.48}$$

according to (2.36). Now $\left(\Gamma_{0123}\beta_0^{(5)}\right)^2 = 1$ obviously and thus $1 \pm \Gamma_{0123}\beta_0^{(5)}$ in the worldvolume Killing spinor equation (2.37) with $p = 5$ play the role of projection operators. Having the projection operators, the next step described in section 2.3 is to send $\Gamma_{0123}\beta_0^{(5)}$ to the right of $S_0(\phi)$ in (2.37). If we denote the resulting expression as $\tilde{\Gamma}$, we get the relation $\Gamma_{0123}\beta_0^{(5)} S_0(\phi) = S_0(\phi)\tilde{\Gamma}$. With the fact that $\Gamma_{0123}\beta_0^{(5)} = -\Gamma_{12346789} = -\Gamma^{05}\Gamma^{11}$ where Γ^{11} of (A.6) has been used, $\tilde{\Gamma}$ is evaluated by repeated use of the identity (2.38) as follows:

$$\begin{aligned} \tilde{\Gamma} &= S_0^{-1}(\phi)\Gamma^{05}\Gamma^{11}S_0(\phi) \\ &= \Big(-\mathring{s}_1\Gamma^{04} + \mathring{c}_1 c_2 \Gamma^{05} + \mathring{c}_1 s_2 s_3 \Gamma^{06} + \mathring{c}_1 s_2 s_3 c_4 \Gamma^{07} \\ &\qquad + \mathring{c}_1 s_2 s_3 s_4 c_5 \Gamma^{08} + \mathring{c}_1 s_2 s_3 s_4 s_5 \Gamma^{09} \Big)\Gamma^{11}\,, \end{aligned} \tag{3.49}$$

where we have used the definitions of eq. (2.42). This shows clearly that $1 \pm \tilde{\Gamma}$ do not have the form of projection operators. One can make them have the desired form by fixing the transverse angular position, and realize that there is a unique choice of $\phi_0^1 = \frac{\pi}{2}$ which leads to $\tilde{\Gamma} = -\Gamma^{04}\Gamma^{11}$. If we now use the chirality property of $\epsilon_\pm$ in eq. (2.14) and split $\epsilon_\pm$ according to the eigenvalues of Γ^{04} as

$$\Gamma^{04}\epsilon_{+\pm} = \pm\epsilon_{+\pm}\,, \quad \Gamma^{04}\epsilon_{-\pm} = \pm\epsilon_{-\pm}\,, \tag{3.50}$$

then the worldvolume Killing spinor equation (2.37) becomes

$$\begin{aligned} 0 = &-2u^{1/2}\Gamma_{0123}S_0(\phi)\left(\epsilon_{+-} - x^0\Gamma_0\epsilon_{-+} - \vec{x}_0\cdot\vec{\Gamma}\epsilon_{--}\right) \\ &+ 2u^{-1/2}\Gamma_{0123}\Gamma_4 S_0(\phi)\epsilon_{-+}\,, \end{aligned} \tag{3.51}$$

with

$$S_0(\phi) = S(\phi)|_{\phi^1=\pi/2}\,. \tag{3.52}$$

The solution of this equation is readily found to be

$$\epsilon_{-+} = 0\,, \quad \epsilon_{+-} = \vec{x}_0\cdot\vec{\Gamma}\epsilon_{--}\,. \tag{3.53}$$

Since other components except for those of (3.53) are undetermined, we conclude that the supersymmetry preserved on the $\mathrm{AdS}_2 \times \mathrm{S}^4$ brane is characterized by

$$\epsilon_{++}\,, \quad \epsilon_{--}\,, \tag{3.54}$$

each of which has eight free components and we have sixteen supersymmetries (1/2-BPS) in total.

Having identified the worldvolume supersymmetries (3.54), it is straightforward to obtain the supersymmetry transformation rules for the worldvolume fields according to (2.40) and (2.41). Firstly, the scalar fields corresponding to the transverse fluctuations are found to transform as

$$\begin{aligned}\delta\tilde{\vec{x}} &= 2iu^{-1}\bar{\theta}\vec{\Gamma}\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \dots\,,\\ \delta\tilde{\phi}^1 &= 2i\bar{\theta}\Gamma_5\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \dots\,.\end{aligned} \tag{3.55}$$

where

$$\begin{aligned}\hat{\eta}_+ &= u^{1/2}S_0(\phi)\left(\epsilon_{++} - x^0\Gamma_0\epsilon_{--}\right)\,,\\ \hat{\eta}_- &= u^{-1/2}\Gamma_4 S_0(\phi)\epsilon_{--}\,.\end{aligned} \tag{3.56}$$

As for the worldvolume gauge field, we obtain

$$\begin{aligned}\delta A_0 &= -2iu\bar{\theta}\Gamma_0\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \dots\,,\\ \delta A_u &= -2iu^{-1}\bar{\theta}\Gamma_4\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \dots\,,\\ \delta A_{\phi^\alpha} &= -2ie^{\hat{a}}_{\phi^\alpha}\bar{\theta}\Gamma_{\hat{a}}\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \dots\,.\end{aligned} \tag{3.57}$$

where $\alpha = 2,3,4,5$ and $\phi^1 = \frac{\pi}{2}$ should be imposed on $e^{\hat{a}}_{\phi^\alpha}$. Finally, we get the transformation rule for the fermionic field as

$$\delta\theta = -2u\tilde{\vec{x}}\cdot\vec{\Gamma}\Gamma_{01234}\hat{\eta}_- + \tilde{\phi}^1\Gamma_5\Gamma_{01234}\left(\hat{\eta}_+ + \hat{\eta}_-\right) - \beta_1^{(5)}\left(\hat{\eta}_+ + \hat{\eta}_-\right) + \dots\,, \tag{3.58}$$

where

$$\begin{aligned}\beta_1^{(5)} = &\left(\frac{1}{u}\Gamma^0\partial_0\tilde{X}^f + u\Gamma^4\partial_u\tilde{X}^f + \Gamma^6\partial_{\phi^2}\tilde{X}^f + \frac{1}{s_2}\Gamma^7\partial_{\phi^3}\tilde{X}^f\right.\\ &\left.+\frac{1}{s_2s_3}\Gamma^8\partial_{\phi^4}\tilde{X}^f + \frac{1}{s_2s_3s_4}\Gamma^9\partial_{\phi^5}\tilde{X}^f\right)e^{\hat{a}}_f\Gamma_{046789\hat{a}}\\ &-\frac{1}{48s_2^3s_3^2s_4}\epsilon^{i_0\cdots i_3i_4i_5}\left(e^{\hat{a}_0}_{\ell_{i_0}}\cdots e^{\hat{a}_3}_{\ell_{i_3}}\right)\Big|_{\phi^1=\pi/2}\Gamma_{\hat{a}_0\cdots\hat{a}_3}F_{i_4i_5}\,.\end{aligned} \tag{3.59}$$

3.4 D7

In this subsection, we are led to consider two kinds of D7-brane configurations. The common content for them is $\beta_\pm^{(7)}$ appearing in the κ symmetry projection $\Gamma^{(7)}$, which is read off from eqs. (2.26) and (2.28) as

$$\begin{aligned}\beta_\pm^{(7)} = \frac{\epsilon^{i_1\cdots i_8}}{\sqrt{-\det\left(G_{ij}+\mathcal{F}_{ij}\right)}}&\left(\pm\frac{1}{8!}\gamma_{i_1\cdots i_8} + \frac{1}{1440}\gamma_{i_1\cdots i_6}\mathcal{F}_{i_7i_8} \pm \frac{1}{192}\gamma_{i_1\cdots i_4}\mathcal{F}_{i_5i_6}\mathcal{F}_{i_7i_8}\right.\\ &\left.+\frac{1}{96}\gamma_{i_1i_2}\mathcal{F}_{i_3i_4}\mathcal{F}_{i_5i_6}\mathcal{F}_{i_7i_8} \pm \frac{1}{384}\mathcal{F}_{i_1i_2}\mathcal{F}_{i_3i_4}\mathcal{F}_{i_5i_6}\mathcal{F}_{i_7i_8}\right)\,.\end{aligned} \tag{3.60}$$

JHEP09(2017)022

3.4.1 $(5,3)$-brane

The D7-brane configuration (5,3) of table 1 leads us to take the static gauge as

$$x^{0,1,2,3}(\sigma) = \sigma^{0,1,2,3}\,, \quad u(\sigma) = \sigma^4\,, \quad \phi^{3,4,5}(\sigma) = \sigma^{5,6,7}\,, \tag{3.61}$$

or $(\ell_0,\ell_1,\ell_2,\ell_3,\ell_4,\ell_5,\ell_6,\ell_7) = (0,1,2,3,4,7,8,9)$ in eq. (2.30), which corresponds to the $\mathrm{AdS}_5\times\mathrm{S}^3$ brane. The coordinates transverse to this configuration are then

$$X^f = \{\phi^1,\phi^2\}\,. \tag{3.62}$$

In the static gauge (3.61), $\beta_-^{(7)}$ of (3.60) becomes,

$$\beta_0^{(7)} = \Gamma_{01234789}\,, \tag{3.63}$$

according to (2.36). Now $\left(\Gamma_{0123}\beta_0^{(7)}\right)^2 = 1$ obviously and thus $1 \pm \Gamma_{0123}\beta_0^{(7)}$ in the worldvolume Killing spinor equation (2.37) with $p=7$ play the role of projection operators. Having the projection operators, the next step described in section 2.3 is to send $\Gamma_{0123}\beta_0^{(7)}$ to the right of $S_0(\phi)$ in (2.37). If we denote the resulting expression as $\tilde\Gamma$, we get the relation $\Gamma_{0123}\beta_0^{(7)}S_0(\phi) = S_0(\phi)\tilde\Gamma$. With the fact that $\Gamma_{0123}\beta_0^{(7)} = -\Gamma_{4789}$, $\tilde\Gamma$ is evaluated by repeated use of the identity (2.38) as follows:

$$\begin{aligned}-\tilde\Gamma &= S_0^{-1}(\phi)\Gamma_{4789}S_0(\phi)\\ &= \left(\mathring{c}_1\Gamma_4 + \mathring{s}_1\mathring{c}_2\Gamma_5 + \mathring{s}_1\mathring{s}_2c_3\Gamma_6 + \mathring{s}_1\mathring{s}_2s_3c_4\Gamma_7 + \mathring{s}_1\mathring{s}_2s_3s_4c_5\Gamma_8 + \mathring{s}_1\mathring{s}_2s_3s_4s_5\Gamma_9\right)\\ &\quad\times\left(c_3\Gamma_{789} - s_3c_4\Gamma_{689} + s_3s_4c_5\Gamma_{679} - s_3s_4s_5\Gamma_{678}\right)\,,\end{aligned} \tag{3.64}$$

where we have used the definitions of eq. (2.42). This shows clearly that $1\pm\tilde\Gamma$ do not have the form of projection operators. One can make them have the desired form by fixing the transverse angular position, and realize that there is a unique choice of $\phi_0^{1,2} = \frac{\pi}{2}$ which leads to $\tilde\Gamma = -\Gamma_{6789}$. If we now split $\epsilon_\pm$ according to the eigenvalues of Γ_{6789} as

$$\Gamma_{6789}\epsilon_{+\pm} = \pm\epsilon_{+\pm}\,, \quad \Gamma_{6789}\epsilon_{-\pm} = \pm\epsilon_{-\pm}\,, \tag{3.65}$$

then the worldvolume Killing spinor equation (2.37) becomes

$$\begin{aligned}0 = &-2u^{1/2}\Gamma_{0123}S_0(\phi)\left(\epsilon_{++} - x\cdot\Gamma\epsilon_{-+}\right)\\ &+2u^{-1/2}\Gamma_{0123}\Gamma_4S_0(\phi)\epsilon_{-+}\,,\end{aligned} \tag{3.66}$$

with

$$S_0(\phi) = S(\phi)|_{\phi^1,\phi^2=\pi/2}\,. \tag{3.67}$$

The solution of this equation is readily found to be

$$\epsilon_{-+} = 0\,, \quad \epsilon_{++} = 0\,. \tag{3.68}$$

Since other components except for those of (3.68) are undetermined, we conclude that the supersymmetry preserved on the $\mathrm{AdS}_5\times\mathrm{S}^3$ brane is characterized by

$$\epsilon_{+-}\,, \quad \epsilon_{--}\,, \tag{3.69}$$

JHEP09(2017)022

each of which has eight free components and we have sixteen supersymmetries (1/2-BPS) in total.

Having identified the worldvolume supersymmetries (3.69), it is straightforward to obtain the supersymmetry transformation rules for the worldvolume fields according to (2.40) and (2.41). Firstly, the scalar fields corresponding to the transverse fluctuations are found to transform as

$$\delta\tilde{\phi}^{1,2} = 2i\bar{\theta}\Gamma^{5,6}\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \dots\,, \tag{3.70}$$

where

$$\begin{aligned}\hat{\eta}_+ &= u^{1/2}S_0(\phi)\left(\epsilon_{+-} - x\cdot\Gamma\epsilon_{--}\right)\,,\\ \hat{\eta}_- &= u^{-1/2}\Gamma_4 S_0(\phi)\epsilon_{--}\,.\end{aligned} \tag{3.71}$$

As for the worldvolume gauge field, we obtain

$$\begin{aligned}\delta A_{0,1,2,3} &= -2iu\bar{\theta}\Gamma_{0,1,2,3}\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \dots\,,\\ \delta A_u &= -2iu^{-1}\bar{\theta}\Gamma_4\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \dots\,,\\ \delta A_{\phi^\alpha} &= -2ie^{\hat{a}}_{\phi^\alpha}\bar{\theta}\Gamma_{\hat{a}}\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \dots\,,\end{aligned} \tag{3.72}$$

where $\alpha = 3,4,5$ and $\phi^{1,2} = \frac{\pi}{2}$ should be imposed on $e^{\hat{a}}_{\phi^\alpha}$. Finally, we get the transformation rule for the fermionic field as

$$\delta\theta = \tilde{\phi}^\alpha\Gamma_{\alpha+4}\Gamma_{01234}\left(\hat{\eta}_+ + \hat{\eta}_-\right) - \beta^{(7)}_1\left(\hat{\eta}_+ + \hat{\eta}_-\right) + \dots\,, \tag{3.73}$$

where $\alpha = 1,2$ and

$$\begin{aligned}\beta^{(7)}_1 = &-\left(\frac{1}{u}\Gamma^0\partial_0\tilde{X}^f + \frac{1}{u}\Gamma^1\partial_1\tilde{X}^f + \frac{1}{u}\Gamma^2\partial_2\tilde{X}^f + \frac{1}{u}\Gamma^3\partial_3\tilde{X}^f + u\Gamma^4\partial_u\tilde{X}^f\right.\\ &\left.+\Gamma^7\partial_{\phi^3}\tilde{X}^f + \frac{1}{s_3}\Gamma^8\partial_{\phi^4}\tilde{X}^f + \frac{1}{s_3 s_4}\Gamma^9\partial_{\phi^5}\tilde{X}^f\right)e^{\hat{a}}_f\Gamma_{01234789\hat{a}}\\ &+\frac{1}{1440u^3 s_3^2 s_4}\epsilon^{i_0\cdots i_5 i_6 i_7}\left(e^{\hat{a}_0}_{\ell_{i_0}}\cdots e^{\hat{a}_5}_{\ell_{i_5}}\right)\Big|_{\phi^1,\phi^2=\pi/2}\Gamma_{\hat{a}_0\cdots\hat{a}_5}F_{i_6 i_7}\,.\end{aligned} \tag{3.74}$$

3.4.2 (3, 5)-brane

The D7-brane configuration (3,5) of table 1 leads us to take the static gauge as

$$x^{0,1}(\sigma) = \sigma^{0,1}\,,\quad u(\sigma) = \sigma^2\,,\quad \phi^{1,\dots,5}(\sigma) = \sigma^{3,\dots,7}\,, \tag{3.75}$$

or $(\ell_0,\ell_1,\ell_2,\ell_3,\ell_4,\ell_5,\ell_6,\ell_7) = (0,1,4,5,6,7,8,9)$ in eq. (2.30), which corresponds to the $\mathrm{AdS}_3\times\mathrm{S}^5$ brane. The coordinates transverse to this configuration are then

$$X^f = \{x^2, x^3\}\,. \tag{3.76}$$

In the static gauge (3.75), $\beta^{(7)}_-$ of (3.60) becomes,

$$\beta^{(7)}_0 = \Gamma_{01456789}\,, \tag{3.77}$$

according to (2.36). Now $\left(\Gamma_{0123}\beta_0^{(7)}\right)^2 = 1$ obviously and thus $1 \pm \Gamma_{0123}\beta_0^{(7)}$ in the worldvolume Killing spinor equation (2.37) with $p = 7$ play the role of projection operators. Having the projection operators, the next step described in section 2.3 is to send $\Gamma_{0123}\beta_0^{(7)}$ to the right of $S_0(\phi)$ in (2.37). But this process is trivial because we see that $\Gamma_{0123}\beta_0^{(7)}$ moves freely to the right of $S_0(\phi)$ by noticing $\Gamma_{0123}\beta_0^{(7)} = \Gamma_{23\ldots9} = \Gamma^{01}\Gamma^{11}$ from the definition of Γ^{11} given in (A.6). If we now split $\epsilon_\pm$ according to the eigenvalues of Γ^{01} as

$$\Gamma^{01}\epsilon_{+\pm} = \pm\epsilon_{+\pm}\,, \quad \Gamma^{01}\epsilon_{-\pm} = \pm\epsilon_{-\pm}\,, \tag{3.78}$$

then the worldvolume Killing spinor equation (2.37) becomes

$$\begin{aligned} 0 = & -2u^{1/2}\Gamma_{0123}S(\phi)\left[\epsilon_{+-} - \left(x^0\Gamma_0 + x^1\Gamma_1\right)\epsilon_{-+} - \left(x_0^2\Gamma_2 + x_0^3\Gamma_3\right)\epsilon_{--}\right] \\ & + 2u^{-1/2}\Gamma_{0123}\Gamma_4 S(\phi)\epsilon_{-+}\,, \end{aligned} \tag{3.79}$$

The solution of this equation is readily found to be

$$\epsilon_{-+} = 0\,, \quad \epsilon_{+-} = \left(x_0^2\Gamma_2 + x_0^3\Gamma_3\right)\epsilon_{--}\,. \tag{3.80}$$

Since other components except for those of (3.80) are undetermined, we conclude that the supersymmetry preserved on the $\mathrm{AdS}_3\times\mathrm{S}^5$ brane is characterized by

$$\epsilon_{++}\,, \quad \epsilon_{--}\,, \tag{3.81}$$

each of which has eight free components and we have sixteen supersymmetries (1/2-BPS) in total.

Having identified the worldvolume supersymmetries (3.81), it is straightforward to obtain the supersymmetry transformation rules for the worldvolume fields according to (2.40) and (2.41). Firstly, the scalar fields corresponding to the transverse fluctuations are found to transform as

$$\delta\tilde{x}^{2,3} = 2iu^{-1}\bar{\theta}\Gamma^{2,3}\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \ldots\,, \tag{3.82}$$

where

$$\begin{aligned} \hat{\eta}_+ &= u^{1/2}S(\phi)\left[\epsilon_{++} - \left(x^0\Gamma_0 + x^1\Gamma_1\right)\epsilon_{--}\right]\,, \\ \hat{\eta}_- &= u^{-1/2}\Gamma_4 S(\phi)\epsilon_{--} \end{aligned} \tag{3.83}$$

As for the worldvolume gauge field, we obtain

$$\begin{aligned} \delta A_{0,1} &= -2iu\bar{\theta}\Gamma_{0,1}\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \ldots\,, \\ \delta A_u &= -2iu^{-1}\bar{\theta}\Gamma_4\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \ldots\,, \\ \delta A_{\phi^\alpha} &= -2ie_{\phi^\alpha}^{\hat{a}}\bar{\theta}\Gamma_{\hat{a}}\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \ldots\,, \end{aligned} \tag{3.84}$$

where $\alpha = 1, \ldots, 5$. Finally, we get the transformation rule for the fermionic field as

$$\delta\theta = -2u\left(\tilde{x}^2\Gamma_2 + \tilde{x}^3\Gamma_3\right)\Gamma_{01234}\hat{\eta}_- - \beta_1^{(7)}\left(\hat{\eta}_+ + \hat{\eta}_-\right) + \ldots\,, \tag{3.85}$$

JHEP09(2017)022

where

$$\beta_1^{(7)} = -\left(\frac{1}{u}\Gamma^0\partial_0\tilde{X}^f + \frac{1}{u}\Gamma^1\partial_1\tilde{X}^f + u\Gamma^4\partial_u\tilde{X}^f + \Gamma^5\partial_{\phi^1}\tilde{X}^f + \frac{1}{s_1}\Gamma^6\partial_{\phi^2}\tilde{X}^f \right.$$
$$\left. + \frac{1}{s_1 s_2}\Gamma^7\partial_{\phi^3}\tilde{X}^f + \frac{1}{s_1 s_2 s_3}\Gamma^8\partial_{\phi^4}\tilde{X}^f + \frac{1}{s_1 s_2 s_3 s_4}\Gamma^9\partial_{\phi^5}\tilde{X}^f\right) e_f^{\hat{a}}\Gamma_{014567890\hat{a}}$$
$$+ \frac{1}{1440 u s_1^4 s_2^3 s_3^2 s_4}\epsilon^{i_0\cdots i_5 i_6 i_7} e_{\ell_{i_0}}^{\hat{a}_0}\cdots e_{\ell_{i_5}}^{\hat{a}_5}\Gamma_{\hat{a}_0\cdots\hat{a}_5}F_{i_6 i_7}\,. \tag{3.86}$$

3.5 Invariance of quadratic action

The transformation rules obtained in the previous subsections are explicit to the leading linear order in the worldvolume fluctuating fields. This means that they can be used to confirm the invariance of the quadratic action coming from the expansion of the gauge fixed D-brane action in terms of the worldvolume fields. In this last subsection, we would like to verify the transformation rules by showing the invariance of the quadratic action. However, we will not consider all the six kinds of AdS branes but take one representative, since the transformation rules have the same pattern.

We consider the (3,1) configuration of D3-brane of section 3.2, the $AdS_3 \times S^1$ brane, as the representative. The D3-brane action is given in eq. (2.15) for $p = 3$ and the closed five-form H_5 in the WZ term [26] is

$$H_5 = -\frac{i}{6}L^{\hat{a}}\wedge L^{\hat{b}}\wedge L^{\hat{c}}\wedge \bar{L}^I \wedge \Gamma^{\hat{a}\hat{b}\hat{c}}\tau_2^{IJ}L^J - i\mathcal{F}\wedge L^{\hat{a}}\wedge \bar{L}^I\wedge\Gamma^{\hat{a}}\tau_1^{IJ}L^J$$
$$+\frac{1}{30}\left(\epsilon^{a_1\cdots a_5}L^{a_1}\wedge\cdots\wedge L^{a_5} + \epsilon^{a_1'\cdots a_5'}L^{a_1'}\wedge\cdots\wedge L^{a_5'}\right)\,. \tag{3.87}$$

Then, in the static gauge of (3.16) and the covariant κ symmetry fixing condition (2.32), the expansion of the action in terms of the fluctuating fields leads us to have the bosonic ($S_B^{(2)}$) and fermionic ($S_F^{(2)}$) parts of the quadratic action as

$$S_B^{(2)} = \int d^4\sigma\sqrt{-g}\Big[-\frac{1}{2}u^2 g^{ij}\partial_i\tilde{x}^m\partial_j\tilde{x}^m - 2u^2\left(\tilde{x}^3\partial_{\phi^5}\tilde{x}^2 - \tilde{x}^2\partial_{\phi^5}\tilde{x}^3\right)$$
$$-\frac{1}{2}g^{ij}\partial_i\tilde{\phi}^\alpha\partial_j\tilde{\phi}^\alpha + \frac{1}{2}\left(\tilde{\phi}^\alpha\right)^2 - \frac{1}{4}g^{ij}g^{kl}F_{ik}F_{jl}\Big]\,,$$
$$S_F^{(2)} = i\int d^4\sigma\sqrt{-g}\left(e_{\bar{a}}^i\bar{\theta}\Gamma^{\bar{a}}\nabla_i\theta + \bar{\theta}\Gamma_{239}\theta\right)\,, \tag{3.88}$$

where $m = 2, 3$, $\alpha = 1, 2, 3, 4$, $\bar{a} = 0, 1, 4, 9$, and the terms linear in the derivative ∂_{ϕ^5} and the fermionic mass term originate from the WZ term. The metric g_{ij} is the induced one on the worldvolume given by

$$g_{ij}d\sigma^i d\sigma^j = -u^2\left(dx^0\right)^2 + u^2\left(dx^1\right)^2 + \frac{du^2}{u^2} + \left(d\phi^5\right)^2\,. \tag{3.89}$$

This induced metric is also expressed as $e_i^{\bar{a}}e_j^{\bar{b}}\eta_{\bar{a}\bar{b}}$, where $e_i^{\bar{a}}$ is defined by the worldvolume field independent part of the pullback of zehnbein (2.23), that is, $e_i^{\bar{a}} \equiv \partial_i X^\mu e_\mu^{\bar{a}}|_{\tilde{X}^f=0}$. In the covariant derivative for the spinor given by $\nabla_i = \partial_i + \frac{1}{4}\omega_i^{\bar{a}\bar{b}}\Gamma_{\bar{a}\bar{b}}$, the spin connection is determined from $e_i^{\bar{a}}$ by using the Cartan structure equation or can be defined, like the definition of $e_i^{\bar{a}}$, as $\omega_i^{\bar{a}\bar{b}} \equiv \partial_i X^\mu \omega_\mu^{\bar{a}\bar{b}}|_{\tilde{X}^f=0}$ from the spacetime spin connection $\omega_\mu^{\bar{a}\bar{b}}$. In the present case, the nonvanishing components are $\omega^{04} = u dx^0$ and $\omega^{14} = u dx^1$.

We note that the Lagrangian density for $\tilde{x}^m$ in the quadratic action (3.88) is not of the canonical form because of the overall u^2 factor. In order to make it canonical, we take the rescaling of $\tilde{x}^m$ as

$$\tilde{x}^m \longrightarrow \frac{\tilde{x}^m}{u}, \tag{3.90}$$

under which the Lagrangian density for the field $\tilde{x}^m$ becomes

$$-\frac{1}{2} g^{ij}\partial_i \tilde{x}^m \partial_j \tilde{x}^m - \frac{3}{2}(\tilde{x}^m)^2 - 2\left(\tilde{x}^3 \partial_{\phi^5}\tilde{x}^2 - \tilde{x}^2\partial_{\phi^5}\tilde{x}^3\right). \tag{3.91}$$

There are also associated changes in the transformation rules of eqs. (3.26) and (3.29) as

$$\begin{aligned}
\delta\tilde{x}^m &= 2i\bar{\theta}\Gamma^m\Gamma_{0123}\left(\hat{\eta}_+ - \hat{\eta}_-\right) + \dots\,, \\
\delta\theta &= -2\tilde{x}^m\Gamma_m\Gamma_{01234}\hat{\eta}_- + \tilde{\phi}^\alpha\Gamma_{\alpha+4}\Gamma_{01234}\left(\hat{\eta}_+ + \hat{\eta}_-\right) - \beta_1^{(3)}\left(\hat{\eta}_+ + \hat{\eta}_-\right) + \dots\,,
\end{aligned} \tag{3.92}$$

where $\tilde{x}^m$ dependent part in $\beta_1^{(3)}$ of (3.30) becomes

$$-e_{\bar{a}}^i\Gamma^{\bar{a}}\partial_i\tilde{x}^m\Gamma_{0149}\Gamma_m + \tilde{x}^m\Gamma_{019}\Gamma_m\,. \tag{3.93}$$

Bearing in mind the above rescaled expressions (3.91), (3.92) and (3.93), we are now ready to consider the transformation of the quadratic action (3.88) by applying the transformation rules (3.26), (3.28) and (3.29) with (3.27) and (3.30). The calculation itself is less trivial but straightforward, and the final result is

$$\delta S_B^{(2)} + \delta S_F^{(2)} = 0\,. \tag{3.94}$$

This invariance of the quadratic action clearly shows that the transformation rules realize the supersymmetry of the worldvolume theory.

4 Non-AdS branes

We turn to the non-AdS branes, in which the AdS radial direction u is transverse to the D-brane configuration and acts as a worldvolume field. From table 1, it turns out that there are six types of Lorentzian non-AdS D-brane configurations, which are supposed to be supersymmetric. For these configurations, the brane embedding coordinates X^{ℓ_i} chosen by the static gauge condition (2.30) and the associated $\beta_0^{(p)}$ of (2.36) are listed in table 2.

As described in section 2.3, a given D-brane configuration is supersymmetric if the eigenvalues of $\Gamma_{0123}\beta_0^{(p)}$ are ± 1, since $1 \pm \Gamma_{0123}\beta_0^{(p)}$ appearing in the worldvolume Killing spinor equation (2.37) are then projection operators and hence used to pick out preserved supersymmetries from ϵ_+ and ϵ_-. For all the expressions of $\beta_0^{(p)}$ listed in table 2, however, $\left(\Gamma_{0123}\beta_0^{(p)}\right)^2 = -1$ which means that the eigenvalues of $\Gamma_{0123}\beta_0^{(p)}$ are $\pm i$. Therefore, we conclude that the static non-AdS D-branes are not supersymmetric.

Dp	(n,n')	X^{ℓ_i}	$\beta_0^{(p)}$
D1	(2,0)	$x^{0,1}$	$-\Gamma_{01}$
D3	(3,1)	$x^{0,1,2}$, ϕ^5	Γ_{0129}
	(1,3)	x^0, $\phi^{3,4,5}$	Γ_{0789}
D5	(4,2)	$x^{0,1,2,3}$, $\phi^{4,5}$	$-\Gamma_{012389}$
	(2,4)	$x^{0,1}$, $\phi^{2,3,4,5}$	$-\Gamma_{016789}$
D7	(3,5)	$x^{0,1,2}$, $\phi^{1,\ldots,5}$	$\Gamma_{01256789}$

Table 2. Configurations of Non-AdS branes.

The radial AdS coordinate in the worldvolume Killing spinor equation (2.37) should be understood as u_0, the AdS radial position of D-brane. One may argue that if a non-AdS D-brane is placed at the origin of u, that is $u_0 = 0$, then the solution of the Killing spinor equation is $\epsilon_- = 0$ while leaving ϵ_+ free and thus the non-AdS brane at such special position is supersymmetric. The problem is that the induced worldvolume metric becomes singular at $u_0 = 0$. To avoid the singularity, one might consider a kind of regularization by introducing a non-vanishing infinitesimal value of u_0. However, he or she encounters again the fact that $1 \pm \Gamma_{0123}\beta_0^{(p)}$ are not projection operators. Thus the conclusion about the supersymmetry of non-AdS branes does not change.

It should be noted that the conclusion in this section is only for the static configurations. If the non-AdS brane takes a certain constant motion along a transverse direction or has non-vanishing fluxes on its worldvolume, the situation may change completely. A typical example is the giant graviton [29–31], one type of which is a (1,3) configuration of D3 brane and takes a constant motion along a transverse angular direction. It is known to be 1/2-BPS for some particular angular speed.

5 Discussion

Starting from the data in table 1, the classification of 1/2-BPS D-branes in the $\mathrm{AdS}_5\times\mathrm{S}^5$ background obtained by the covariant open string description, we have considered all possible static D-brane configurations without worldvolume fluxes. We have identified which part of the target spacetime supersymmetry is preserved on the D-brane worldvolume by solving the worldvolume Killing spinor equation and showed that only the AdS type D-branes where the AdS radial direction is a worldvolume coordinate are 1/2-BPS. As for the supersymmetric configurations, we have obtained the associated worldvolume supersymmetry transformation rules for the worldvolume fields.

One interesting point in the study of supersymmetric configuration is that the transverse angular position has been determined without resort to the equations of motion. The position is fixed only by requiring the worldvolume supersymmetry and sometimes the non-degeneracy of induced worldvolume metric. For example, let us consider the $\mathrm{AdS}_4\times\mathrm{S}^2$ embedding of D5-brane of section 3.3.1, which was also explored in ref. [5] related to the holographic description of the defect conformal field theory [4]. As shown in [5], there are two solutions of the equations of motion for the transverse angular position and it turns out that only one of them leads to the supersymmetric configuration which is the same with the angular position determined in this paper as it should be. In fact, this kind of situation seems to be natural. Usually, the solution of the Killing spinor equation satisfies also the equations of motion. Thus the fact that the transverse angular position is determined in the process of solving the Killing spinor equation may not be surprising.

As shown in section 4, all the static non-AdS branes without any worldvolume fluxes are not supersymmetric. We emphasize again that this may change completely when there are motions in transverse directions or the worldvolume fluxes are turned on, since we already know at least one definite example, the giant graviton. Beyond the static case, there will be lots of possibilities. Having said that, we expect that there will be a suitable classification facilitating the study of them similar to the static D-brane configurations classified as the AdS and non-AdS types.

Finally, we would like to comment on a specific brane configuration, which is the D3-brane spanning the $x^{0,1,2,3}$ directions. If we consider the static configuration

$$\sigma^\mu = x^\mu\,, \quad u = \text{constant}\,, \quad \phi^\alpha = \text{constant}\,, \tag{5.1}$$

where $\mu = 0,1,2,3$ and $\alpha = 1,\ldots,5$, this satisfies the D3-brane equation of motion. As discussed in [34], the corresponding D3 brane action vanishes so that it satisfies 'no force' condition. This is (4,0) brane in our notation. However as shown by [8], neither the usual covariant gauge fixing nor Killing spinor gauge fixing works to obtain supersymmetric world volume gauge theories. This configuration corresponds to D3 brane parallel to N D3 branes, which are geometrized as we go to the near horizon limit. On the other hand, all brane configurations we consider come from intersecting brane configurations, where the other D brane further breaks the half of the supersymmetry of the D3 branes. Interestingly these intersecting brane configurations precisely match the 1/2-BPS configurations classified by covariant open string analysis [17–19]. For theses configurations, the usual covariant κ gauge fixing has no problem.

In [34], the authors consider a different gauge to attack the κ gauge fixing problem of the parallel D3 brane. With $\bar\gamma = \epsilon^{IJ}\Gamma_{0123}$, they define

$$\theta \equiv (1-\bar\gamma)\,\Theta \ \ , \eta \equiv (1+\bar\gamma)\,\Theta \tag{5.2}$$

where indices I, J of Θ being suppressed. Setting $\theta = 0$ corresponds to the usual Killing gauge, which does not work for the parallel D3 brane case. Instead they choose $\eta = 0$, which was shown to lead to the sensible kappa-gauge fixing for the D3 brane action. Thus the treatment of this parallel brane configuration should be rather different from the other configurations considered in the current paper.

One possible way to obtain the four dimensional supersymmetric world volume theory is to start from the $(4,2)$-brane of table 1 and carry out the dimensional reduction. Let us consider the non-AdS type D5-brane which spans $x^{0,1,2,3}$ directions and wraps S^2 inside S^5. If we take the limit of shrinking S^2, we get an effective D3-brane. However, as shown in the last section, such non-AdS type brane is not supersymmetric. In order to make the configuration supersymemtric, it is thus natural to consider turning on some worldvolume flux. Since we would like to keep the Poincaré invariance along $x^{0,1,2,3}$ directions, we turn on flux on S^2. Then we can check the supersymmetry of the configuration after taking the limit of shrinking S^2. Indeed, the resulting effective D3-brane has been shown to be 1/2-BPS for some particular value of flux [5]. In this way, we can have four dimensional supersymmetric worldvolume theory. It would be very interesting to construct such theory and compare its supersymmetry structure with that explored in [34].

Acknowledgments

This work was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT and Future Planning with Grant No. NRF-2015R1A2A2A01007058 (JP) and NRF-2015R1A2A2A01004532 (HS).

A Notation and convention

The notation for the supercoordinate we use is

$$Z^M = \left(X^\mu, \Theta^I\right) , \tag{A.1}$$

where the spinor index for the fermionic coordinate Θ has been suppressed, μ is the ten-dimensional curved space-time vector index, and I $(= 1, 2)$ is introduced to distinguish the two spinors with the same chirality (The chirality is taken to be positive). As for the Lorentz frame or the tangent space, the vector index is denoted by

$$\hat{a} = (a, a') , \quad a = 0, 1, 2, 3, 4 , \quad a' = 5, \dots, 9 , \tag{A.2}$$

where a (a') corresponds to the tangent space of AdS_5 (S^5), and the metric $\eta_{\hat{a}\hat{b}}$ follows the most plus sign convention as $\eta_{\hat{a}\hat{b}} = \mathrm{diag}(-, +, +, \dots, +)$.

The matrices acting on the spinors indexed with $I, J, \dots$ are denoted by

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \tau_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} , \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \tag{A.3}$$

The explicit expression for the vector (spinor) superfield or the Maurer-Cartan one-form superfield $L^{\hat{a}} = dZ^M L^{\hat{a}}_M$ $(L^I = dZ^M L^I_M)$ is given by [32, 33]

$$\begin{aligned} L^{\hat{a}} &= e^{\hat{a}} - 2i \sum_{n=0}^{15} \frac{1}{(2n+2)!} \bar{\Theta}^I \Gamma^{\hat{a}} \left(\mathcal{M}^{2n}\right)^{IJ} D\Theta^J , \\ L^I &= \sum_{n=0}^{16} \frac{1}{(2n+1)!} \left(\mathcal{M}^{2n}\right)^{IJ} D\Theta^J , \end{aligned} \tag{A.4}$$

where $\Gamma^{\hat{a}}$ is the 32×32 Dirac gamma matrix, and $\mathcal{M}^2$ and the spinor covariant derivative $D\Theta^I$ are, in the 32 component notation,

$$\begin{aligned} (\mathcal{M}^2)^{IJ} &= -\epsilon^{IK} \Gamma_* \Gamma^{\hat{a}} \Theta^K \bar{\Theta}^J \Gamma_{\hat{a}} + \frac{1}{2} \tau_2{}^{KJ} \left(\Gamma^{ab} \Theta^I \bar{\Theta}^K \Gamma_{ab} \Gamma_* - \Gamma^{a'b'} \Theta^I \bar{\Theta}^K \Gamma_{a'b'} \Gamma'_*\right) , \\ D\Theta^I &= \left(d + \frac{1}{4} \omega^{\hat{a}\hat{b}} \Gamma_{\hat{a}\hat{b}}\right) \Theta^I - \frac{i}{2} \tau_2{}^{IJ} e^{\hat{a}} \Gamma_* \Gamma_{\hat{a}} \Theta^J . \end{aligned} \tag{A.5}$$

Some definitions of gamma matrix products and their properties are as follows:

$$\Gamma_* \equiv i\Gamma^{01234}\,, \qquad \Gamma'_* \equiv -i\Gamma^{56789}\,, \qquad \Gamma_*^2 = 1\,, \quad \Gamma_*'^2 = -1\,,$$
$$\Gamma^{11} = \Gamma^{01\dots 9} = \Gamma_*\Gamma'_*\,, \qquad (\Gamma^{11})^2 = 1\,. \tag{A.6}$$

References

[1] J.M. Maldacena, *The Large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 23 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200] [INSPIRE].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from noncritical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109] [INSPIRE].

[3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150] [INSPIRE].

[4] O. DeWolfe, D.Z. Freedman and H. Ooguri, *Holography and defect conformal field theories*, *Phys. Rev.* **D 66** (2002) 025009 [hep-th/0111135] [INSPIRE].

[5] K. Skenderis and M. Taylor, *Branes in AdS and p p wave space-times*, *JHEP* **06** (2002) 025 [hep-th/0204054] [INSPIRE].

[6] M. Aganagic, C. Popescu and J.H. Schwarz, *D-brane actions with local kappa symmetry*, *Phys. Lett.* **B 393** (1997) 311 [hep-th/9610249] [INSPIRE].

[7] M. Aganagic, C. Popescu and J.H. Schwarz, *Gauge invariant and gauge fixed D-brane actions*, *Nucl. Phys.* **B 495** (1997) 99 [hep-th/9612080] [INSPIRE].

[8] H. Shin, *On κ symmetry fixing of the D3 brane in $AdS_5 \times S^5$*, *J. Korean Phys. Soc.* **67** (2015) 433.

[9] J.F.G. Cascales and A.M. Uranga, *Branes on generalized calibrated submanifolds*, *JHEP* **11** (2004) 083 [hep-th/0407132] [INSPIRE].

[10] O. de Felice and J. Geipel, *Generalised Calibrations in AdS backgrounds from Exceptional Sasaki-Einstein Structures*, arXiv:1704.05949 [INSPIRE].

[11] P. Pasti, D.P. Sorokin and M. Tonin, *Branes in superAdS backgrounds and superconformal theories*, hep-th/9912076 [INSPIRE].

[12] N.D. Lambert and P.C. West, *D-branes in the Green-Schwarz formalism*, *Phys. Lett.* **B 459** (1999) 515 [hep-th/9905031] [INSPIRE].

[13] P. Bain, K. Peeters and M. Zamaklar, *D-branes in a plane wave from covariant open strings*, *Phys. Rev.* **D 67** (2003) 066001 [hep-th/0208038] [INSPIRE].

[14] S.-j. Hyun, J. Park and H.-j. Shin, *Covariant description of D-branes in IIA plane wave background*, *Phys. Lett.* **B 559** (2003) 80 [hep-th/0212343] [INSPIRE].

[15] M. Blau, J.M. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *Penrose limits and maximal supersymmetry*, *Class. Quant. Grav.* **19** (2002) L87 [hep-th/0201081] [INSPIRE].

[16] D.Z. Freedman, K. Skenderis and M. Taylor, *World volume supersymmetries for branes in plane waves*, *Phys. Rev.* **D 68** (2003) 106001 [hep-th/0306046] [INSPIRE].

[17] M. Sakaguchi and K. Yoshida, *D-branes of covariant AdS superstrings*, *Nucl. Phys.* **B 684** (2004) 100 [hep-th/0310228] [INSPIRE].

[18] M. Sakaguchi and K. Yoshida, *Notes on D-branes of type IIB string on* $AdS_5 \times S^5$, *Phys. Lett.* **B 591** (2004) 318 [hep-th/0403243] [INSPIRE].

[19] E. Chang-Young, H. Nakajima and H. Shin, *Consistent bilinear Wess-Zumino term for open AdS superstring*, *JHEP* **10** (2012) 048 [arXiv:1207.7210] [INSPIRE].

[20] S. Hanazawa and M. Sakaguchi, *D-branes from pure spinor superstring in* $AdS_5\times S^5$ *background*, *Nucl. Phys.* **B 914** (2017) 234 [arXiv:1609.05457] [INSPIRE].

[21] N. Berkovits, *Super Poincaré covariant quantization of the superstring*, *JHEP* **04** (2000) 018 [hep-th/0001035] [INSPIRE].

[22] P. Claus and R. Kallosh, *Superisometries of the AdS x S superspace*, *JHEP* **03** (1999) 014 [hep-th/9812087] [INSPIRE].

[23] H. Lü, C.N. Pope and J. Rahmfeld, *A Construction of Killing spinors on S**n*, *J. Math. Phys.* **40** (1999) 4518 [hep-th/9805151] [INSPIRE].

[24] M. Sakaguchi and K. Yoshida, *Non-relativistic AdS branes and Newton-Hooke superalgebra*, *JHEP* **10** (2006) 078 [hep-th/0605124] [INSPIRE].

[25] E. Bergshoeff, R. Kallosh, T. Ortín and G. Papadopoulos, *Kappa symmetry, supersymmetry and intersecting branes*, *Nucl. Phys.* **B 502** (1997) 149 [hep-th/9705040] [INSPIRE].

[26] R.R. Metsaev and A.A. Tseytlin, *Supersymmetric D3-brane action in* $AdS_5 \times S^5$, *Phys. Lett.* **B 436** (1998) 281 [hep-th/9806095] [INSPIRE].

[27] R.R. Metsaev, *Supersymmetric D3-brane and* $N = 4$ *SYM actions in plane wave backgrounds*, *Nucl. Phys.* **B 655** (2003) 3 [hep-th/0211178] [INSPIRE].

[28] A. Karch and L. Randall, *Open and closed string interpretation of SUSY CFT's on branes with boundaries*, *JHEP* **06** (2001) 063 [hep-th/0105132] [INSPIRE].

[29] J. McGreevy, L. Susskind and N. Toumbas, *Invasion of the giant gravitons from Anti-de Sitter space*, *JHEP* **06** (2000) 008 [hep-th/0003075] [INSPIRE].

[30] M.T. Grisaru, R.C. Myers and O. Tafjord, *SUSY and goliath*, *JHEP* **08** (2000) 040 [hep-th/0008015] [INSPIRE].

[31] A. Hashimoto, S. Hirano and N. Itzhaki, *Large branes in AdS and their field theory dual*, *JHEP* **08** (2000) 051 [hep-th/0008016] [INSPIRE].

[32] R.R. Metsaev and A.A. Tseytlin, *Type IIB superstring action in* $AdS_5 \times S^5$ *background*, *Nucl. Phys.* **B 533** (1998) 109 [hep-th/9805028] [INSPIRE].

[33] R. Kallosh, J. Rahmfeld and A. Rajaraman, *Near horizon superspace*, *JHEP* **09** (1998) 002 [hep-th/9805217] [INSPIRE].

[34] P. Pasti, D.P. Sorokin and M. Tonin, *On gauge fixed superbrane actions in AdS superbackgrounds*, *Phys. Lett.* **B 447** (1999) 251 [hep-th/9809213] [INSPIRE].

JHEP09(2017)022