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A Control Chart for Gamma Distribution using Multiple Dependent State Sampling

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ABSTRACT

In this article, a control chart based on multiple dependent (or deferred) state sampling for the gamma distributed quality characteristic is proposed using the gamma to normal transformation. The proposed control chart has two pairs of control limits, which can be determined by considering the in-control average run length (ARL). The shift in the scale parameter of a gamma distribution is considered and the out-of-control ARL is evaluated. The performance of the proposed chart has been shown for different levels of the parameters of the proposed control chart. It is also shown that the proposed chart is better than the Shewhart chart in terms of ARLs. A case study with a real data has been included for the practical usage of the proposed scheme.

Keywords: Multiple Dependent states, Control Chart, Gamma Distribution, Wilson-Hilferty Transformation, Average Run Length, Simulation

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1. INTRODUCTION

A control chart is an important tool of statistical process control for monitoring and improving the quality of products of any manufacturing process. The idea of control chart was rooted by Shewhart A. Walter during 1920s in Bell Telephone Laboratories. Several modifications have been introduced since its existence but the basic idea of plotting the statistic on the graph of lower and upper limits remains unchanged. It becomes necessary for quality engineers to evaluate the control chart in use whether it has the ability of early detection of the out-of-control process. Early and quick detection of the assignable causes of the on-line process is the prime purpose behind constructing the control charts.

The concept of multiple dependent (or deferred) state (MDS) sampling was initiated by (Wortham and Baker, 1976). Balamurali and Jun (2007) presented a variable acceptance sampling plan using the MDS scheme and concluded that this sampling scheme is better in risk protection to the manufacturer and the consumer as compared to the conventional single and double sampling plans. (Aslam *et al.*, 2015) studied the MDS schemes in the area of acceptance sampling plans and argued that the MDS sampling performs better than the conventional single sampling plans in terms of average sample number. Under the MDS scheme the decision about the in-control or the out-of-control process is made considering the results of the previous samples. If we select a sample from the on line process and posted it on the control chart, then it may fall in any of three mutually exclusive states i.e., in-control state, out-of-control state or the state in which the decision depends on the previous samples. The MDS sampling have been studied by many authors including among others (Soundararajan and Vijayaraghavan, 1990).

Most of the control charts have been studied assuming that the specific quality characteristics of the manufacturing process follow the normal distribution. But there are situations when the specific quality characteristic does not follow the normal distribution. According to Santiago and Smith (2013) the data not collected in subgroups or a skewed data may not produce good results under the normal distribution. Schilling and Nelson (1976) and Stoumbos and Reynolds Jr (2000) have suggested alternative methods when the quality characteristic of interest follows a skewed distribution. Santiago and Smith (2013) proposed control chart for exponential distribution and named it as t-chart. Santiago and Smith (2013) used transformation given by Johnson and Kotz (1970) and Nelson (1994). Mohammed (2004) and Mohammed and Laney (2006) applied t-chart in healthcare. Aslam *et al*. (2016) proposed t-chart using process capability index. For a skewed distributed quality characteristic, a popularly used distribution to study the phenomena is a gamma distribution. The gamma distribution is frequently used in modeling the waiting time of the life events (Hogg and Craig, 1970; Aksoy, 2000). Al-Oraini and Rahim (2002) worked for economical X-bar chart for gamma distribution. Jearkpaporn *et al*. (2003) designed control chart for gamma distribution using generalized linear model. Sheu and Lin (2003) used the gamma distribution to study a small shift in the process. Aslam *et al*. (2014) used the Wilson-Hilferty transformation to propose a control chart for an exponential distribution. Zhang *et al*. (2007) proposed control chart for gamma distribution.

By exploring the literature and according to the best of author knowledge, there is no work on the designing the control chart for a gamma distribution using MDS sampling. Therefore, this study proposes a new control chart for a gamma distribution using MDS sampling. The rest of the paper is organized as follows: In Section 2 the design of the proposed control chart has been explained. Section 3 explains the performance evaluation of the proposed chart in terms of the average run lengths. In Section 4 the comparison of the proposed chart with the Shewhart chart has been described and a simulation study is performed to demonstrate the merit of the proposed control chart. A case study with a real data is also added in this section. In Section 5 some conclusions and findings have been explained.

2. DESIGN OF PROPOSED CONTROL CHART

The proposed control chart utilizes a gamma to normal approximation under the Wilson-Hilferty transformation. Let T be a random variable from a gamma distribution with shape parameter 'a' and scale parameter 'b'. The cumulative distribution function (cdf) of the gamma distribution is given by

$$P\left(T \le t\right) = 1 - \sum_{j=1}^{a-1} \frac{e^{-\frac{t}{b}} \left(t/b \right)^{j}}{j!} \tag{1}$$

The Wilson and Hilferty (1931) suggested that the transformed variable of $T^* = T^{1/3}$ is distributed approximately as normal with mean

$$\mu_{T^*} = \frac{b^{1/3}\Gamma(a+1/3)}{\Gamma(a)} \tag{2}$$

and variance

$$\sigma_{T^*} = \frac{b^{2/3}\Gamma(a+2/3)}{\Gamma(a)} - \mu_{T^*}^2 \tag{3}$$

This suggests that T^* is symmetric in distribution, so a control chart can be designed with the usual upper control limit (UCL) and lower control limit (LCL). Therefore, we propose the following steps for the development of the control chart for a gamma distributed quality characteristic:

Step 1: Select an item randomly and measure its quality characteristic T. Then, calculate T^*:

$$T^* = T^{1/3}$$

Step 2: Declare the process as in-control if $LCL_2 \le T^* \le UCL_2$. Declare the process to be out-of-control if $T^* \ge UCL_1$ or $T^* \le LCL_1$. Otherwise, go to Step-3.

Step-3: Declare the process is in-control if i proceeding subgroups have been declared as in-control. Otherwise, declare the process to be out-of-control.

The proposed control chart is based on two pairs of control limits, that is the outer control limits of (LCL_1, UCL_1) and the inner control limits of (LCL_2, UCL_2) as well as the parameter i. The outer control limits are given by

$$LCL_1 = \mu_{T^*} - k_1\sigma_{T^*} = \frac{b_0^{1/3}\Gamma(a+1/3)}{\Gamma(a)} - k_1\sqrt{\frac{b_0^{2/3}\Gamma(a+2/3)}{\Gamma(a)} - \mu_{T^*}^2} \tag{4a}$$

$$UCL_1 = \mu_{T^*} + k_1\sigma_{T^*} = \frac{b_0^{1/3}\Gamma(a+1}{\Gamma(a)} + k_1\sqrt{\frac{b_0^{2/3}\Gamma(a+2/3)}{\Gamma(a)} - \mu_{T^*}^2} \tag{4b}$$

Also, the inner control limits are given by

$$LCL_2=\mu_{T^*}-k_2\sigma_{T^*}=\frac{b_0^{1/3}\Gamma(a+1/3)}{\Gamma(a)}-k_2\sqrt{\frac{b_0^{2/3}\Gamma(a+2/3)}{\Gamma(a)}-\mu_{T^*}^2} \tag{5a}$$

$$UCL_2=\mu_{T^*}+k_2\sigma_{T^*}=\frac{b_0^{1/3}\Gamma(a+1/3)}{\Gamma(a)}+k_2\sqrt{\frac{b_0^{2/3}\Gamma(a+2/3)}{\Gamma(a)}-\mu_{T^*}^2} \tag{5b}$$

In above, k_1 and k_2 are control coefficient to be determined by considering the in-control ARLs while b_0 is the scale parameter when the process is in control. The proposed plan reduces to the traditional Shewhart control chart when the control coefficients $k_1=k_2=k$ and $i=1$.

The control limits can also be written as follows

$$LCL_1=b_0^{1/3}LL_1$$
$$UCL_1=b_0^{1/3}UL_1$$
$$LCL_2=b_0^{1/3}LL_2$$

and

$$UCL_2=b_0^{1/3}UL_2$$

where

$$LL_1=\left[\frac{\Gamma(a+1/3)}{\Gamma(a)}-k_1\sqrt{\frac{\Gamma(a+2/3)}{\Gamma(a)}-\left(\frac{\Gamma(a+1/3)}{\Gamma(a)}\right)^2}\right] \tag{6a}$$

$$UL_1=\left[\frac{\Gamma(a+1/3)}{\Gamma(a)}+k_1\sqrt{\frac{\Gamma(a+2/3)}{\Gamma(a)}-\left(\frac{\Gamma(a+1/3)}{\Gamma(a)}\right)^2}\right] \tag{6b}$$

$$LL_2=\left[\frac{\Gamma(a+1/3)}{\Gamma(a)}-k_2\sqrt{\frac{\Gamma(a+2/3)}{\Gamma(a)}-\left(\frac{\Gamma(a+1/3)}{\Gamma(a)}\right)^2}\right] \tag{7a}$$

and

$$UL_2=\left[\frac{\Gamma(a+1/3)}{\Gamma(a)}+k_2\sqrt{\frac{\Gamma(a+2/3)}{\Gamma(a)}-\left(\frac{\Gamma(a+1/3)}{\Gamma(a)}\right)^2}\right] \tag{7b}$$

The probability of declaring as in-control for the proposed control chart when the process is actually in control is given as follows

$$P_{in}^0=P\left(LCL_2\le T^*\le UCL_2|b_0\right)+\left\{P\left(LCL_1<T^*<LCL_2\mid b_0\right)+P\left(UCL_2<T^*<UCL_1\mid b_0\right)\right\}\left\{P\left(LCL_2<T^*<UCL_2\mid b_0\right)\right\}^i \tag{8}$$

Here,

$$P\left(LCL_2\le T^*\le UCL_2\mid b_0\right)=P\left(T^*<UCL_2\mid b_0\right)-P\left(T^*<LCL_2\mid b_0\right)$$

$$P\left(LCL_2\le T^*\le UCL_2\mid b_0\right)=\sum_{j=1}^{a-1}\frac{e^{-UL_2^3}(UL_2^3)^j}{j!}-\sum_{j=1}^{a-1}\frac{e^{-LL_2^3}(LL_2^3)^j}{j!}$$

$$P\left(LCL_1<T^*<LCL_2\mid b_0\right)=\sum_{j=1}^{a-1}\frac{e^{-LL_2^3}(UL_2^3)^j}{j!}-\sum_{j=1}^{a-1}\frac{e^{-LL_1^3}(LL_1^3)^j}{j!}$$

$$P\left(UCL_2<T^*<UCL_1\mid b_0\right)=\sum_{j=1}^{a-1}\frac{e^{-UL_1^3}(UL_1^3)^j}{j!}-\sum_{j=1}^{a-1}\frac{e^{-UL_2^3}(UL_2^3)^j}{j!}$$

The average run length (ARL) for the in-control process is given as follows

$$ARL_0=\frac{1}{1-P_{in}^0} \tag{9}$$

Now, we will work for the shifted process. We assumed that the scale parameter of the gamma distribution is shifted from b_0 to b_1 when the process is shifted. Let us assume that $b_1=cb_0$, where b_1 is the shifted scale parameter of the gamma distribution and c is the shift constant. Then, the probability of declaring in-control when the process is shifted is given by $P_{in}^1=P\left(LCL_2\le T^*\le UCL_2\mid b_1\right)+\left\{P\left(LCL_1<T^*<LCL_2\mid b_1\right)+P\left(UCL_2<T^*<UCL_1\mid b_1\right)\right\}\left\{P\left(LCL_2\le T^*\le UCL_2\mid b_1\right)\right\}^i$

Here,

$$P\left(LCL_2\le T^*\le UCL_2\mid b_1\right)=P\left(T^*<UCL_2\mid b_1\right)-P\left(T^*<LCL_2\mid b_1\right)$$

$$P\left(LCL_2\le T^*\le UCL_2\mid b_1\right)=\sum_{j=1}^{a-1}\frac{e^{\frac{-UL_2^3}{c}}(\frac{UL_2^3}{c})^j}{j!}-\sum_{j=1}^{a-1}\frac{e^{-\frac{LL_2^3}{c}}(\frac{LL_2^3}{c})^j}{j!}$$

$$P\left(LCL_1<T^*<LCL_2\mid b_1\right)=\sum_{j=1}^{a-1}\frac{e^{\frac{-LL_2^3}{c}}(\frac{UL_2^3}{c})^j}{j!}$$

$$-\sum_{j=1}^{a-1}\frac{e^{-\frac{LL_1^3}{c}}(\frac{LL_1^3}{c})^j}{j!}$$

$$P\left(UCL_2 < T^* < UCL_1 \mid b_1\right)=\sum_{j=1}^{a-1}\frac{e^{\frac{-UL_1^3}{c}}(\frac{UL_1^3}{c})^j}{j!}$$

$$-\sum_{j=1}^{a-1}\frac{e^{-\frac{UL_2^3}{c}}(\frac{UL_2^3}{c})^j}{j!}$$

The ARL for the shifted process ARL_1 is given as follows

$$ARL_1 = \frac{1}{1-P_{in}^1}$$

3. PERFORMANCE EVALUATION OF THE PROPOSED CHART

The performance indicator of any control chart can be best examined and evaluated by the average run length (ARL). Traditionally, the ARL is defined as the average number of samples before the process shows an out-of-control signal (Montgomery, 2007). A greater value of ALR is required when the process is stable and a smaller value is desirable when the process is shifted or out of control. The simulation approach has been used for estimating the ARL with the help of the R-language software. This simulation approach is commonly used when the exact form of the mean and other measures of the proposed process is not available. Many researchers have used the simulation approach for the effectiveness of control charts including among others Santos (2009), Abbasi and Miller (2013), Ahmad *et al*. (2013), Ahmad *et al*. (2013), Chananet *et al*. (2014), Shu *et al*. (2014), Aslam *et al*. (2015), Azam *et al*. (2015) and Aslam (2016).

The ARL_1 values of the shifted process for r_0 = 200, 300 and 370, for different shift levels c and five values of the shape parameter a = 1, 2, 5, 10 and 20 are given in Table 1-Table 6. Table 1-Table 3 are for I = 2 and Table 4-Table 6 are for i = 3. As mentioned earlier that the shift occurs in the scale parameter as $b_1 = cb_0$ when all other settings are held constants, the decreasing pattern of the ARL_1 shows the performance of the proposed chart. From Table 1~Table 6, we note following trends in control chart parameters.

1. For all other same parameters, as r_0 increases from 200 to 370, the values of ARL_1 increases.
2. For all other same parameters, as a increases from 2 to 10, the values of ARL_1 decreases.
3. For all other same parameters, as i increases from 2 to 3, the values of ARL_1 decreases.

Table 1. The ARL_1 values for the proposed chart with i = 2 when $r_0 = 200$

c	a = 2	a = 5	a = 10	a = 20
	$k_1 = 3.2826$	$k_1 = 4.4368$	$k_1 = 5.6645$	$k_1 = 7.2235$
	$k_2 = 2.8863$	$k_2 = 4.0851$	$k_2 = 5.1897$	$k_2 = 6.7783$
1.00	200.25	200.00	200.00	200.08
1.01	187.12	182.24	175.85	168.34
1.02	175.09	166.40	155.09	142.29
1.03	164.05	152.23	137.19	120.82
1.04	153.90	139.53	121.71	103.04
1.05	144.55	128.13	108.28	88.27
1.10	107.55	85.86	62.83	43.34
1.15	82.16	59.84	38.78	23.46
1.20	64.22	43.17	25.30	13.87
1.30	41.63	24.54	12.42	6.09
1.40	28.80	15.37	7.14	3.41
1.50	20.99	10.42	4.65	2.30
1.60	15.97	7.53	3.34	1.75
1.70	12.59	5.73	2.59	1.47
1.80	10.22	4.56	2.12	1.30
1.90	8.51	3.75	1.82	1.20
2.00	7.23	3.19	1.62	1.13
2.50	4.02	1.88	1.19	1.02
3.00	2.82	1.45	1.07	1.00

Table 2. The ARL_1 values for the proposed chart with i = 2 when $r_0 = 300$

c	$a = 2$	$a = 5$	$a = 10$	$a = 20$
	$k_1 = 3.3913$	$k_1 = 4.59934$	$k_1 = 5.78228$	$k_1 = 7.2915$
	$k_2 = 3.0756$	$k_2 = 3.978001$	$k_2 = 5.207946$	$k_2 = 6.872473$
	ARL1s	*ARL1s*	*ARL1s*	*ARL1s*
1.00	300.08	300.11	300.28	300.90
1.01	279.54	269.73	261.07	252.13
1.02	260.78	242.99	227.74	212.23
1.03	243.62	219.40	199.31	179.44
1.04	227.89	198.54	174.98	152.38
1.05	213.46	180.04	154.11	129.95
1.10	156.73	113.84	85.34	62.30
1.15	118.30	75.39	50.58	32.83
1.20	91.46	52.02	31.85	18.85
1.30	58.13	27.52	14.77	7.80
1.40	39.51	16.36	8.14	4.14
1.50	28.34	10.67	5.13	2.65
1.60	21.24	7.50	3.60	1.95
1.70	16.51	5.61	2.73	1.58
1.80	13.23	4.40	2.21	1.37
1.90	10.88	3.59	1.88	1.25
2.00	9.14	3.03	1.66	1.16
2.50	4.84	1.79	1.20	1.03
3.00	3.27	1.40	1.07	1.00

Table 3. The ARL_1 values for the proposed chart with i = 2 when $r_0 = 370$

c	$a=2$	$a=5$	$a=10$	$a=20$
	$k_1 = 3.470263$	$k_1 = 4.621132$	$k_1 = 5.862599$	$k_1 = 7.385778$
	$k_2 = 2.963487$	$k_2 = 4.078435$	$k_2 = 5.197681$	$k_2 = 6.807985$
1.00	370.02	370.01	370.30	370.17
1.01	343.07	332.80	319.30	304.56
1.02	318.56	300.00	276.32	251.92
1.03	296.23	271.03	239.98	209.46
1.04	275.85	245.38	209.14	175.05
1.05	257.23	222.62	182.89	147.04
1.10	184.89	140.90	98.13	66.08
1.15	136.83	93.25	56.67	33.22
1.20	103.89	64.20	34.95	18.47
1.30	63.95	33.69	15.71	7.38
1.40	42.31	19.80	8.47	3.88
1.50	29.68	12.75	5.27	2.49
1.60	21.84	8.84	3.65	1.85
1.70	16.72	6.52	2.76	1.52
1.80	13.23	5.05	2.22	1.33
1.90	10.77	4.07	1.88	1.21
2.00	8.97	3.40	1.66	1.14
2.50	4.64	1.92	1.19	1.02
3.00	3.12	1.46	1.07	1.00

Table 4. The ARL_1 values for the proposed chart with i = 3 when $r_0 = 200$

c	$a=2$	$a=5$	$a=10$	$a=20$
	$k_1 = 3.282016$	$k_1 = 4.48039$	$k_1 = 5.655759$	$k_1 = 7.28655$
	$k_2 = 2.944851$	$k_2 = 3.995296$	$k_2 = 5.262485$	$k_2 = 6.751103$
1.00	200.01	200.03	200.06	200.05
1.01	186.87	180.98	176.05	165.75
1.02	174.83	164.10	155.38	138.08
1.03	163.78	149.10	137.54	115.65
1.04	153.62	135.76	122.09	97.37
1.05	144.27	123.85	108.67	82.41
1.10	107.24	80.51	63.14	38.55
1.15	81.84	54.64	38.97	20.25
1.20	63.91	38.54	25.40	11.80
1.30	41.34	21.19	12.44	5.20
1.40	28.54	13.02	7.16	3.00
1.50	20.77	8.74	4.67	2.09
1.60	15.78	6.31	3.37	1.65
1.70	12.43	4.83	2.62	1.41
1.80	10.08	3.88	2.16	1.27
1.90	8.39	3.23	1.86	1.18
2.00	7.13	2.77	1.66	1.12
2.50	3.99	1.73	1.21	1.02
3.00	2.81	1.38	1.08	1.00

Table 5. The ARL_1 values for the proposed chart with i = 3 when $r_0 = 300$

c	$a=2$	$a=5$	$a=10$	$a=20$
	$k_1 = 3.409297$	$k_1 = 4.585376$	$k_1 = 5.862102$	$k_1 = 7.387062$
	$k_2 = 2.988333$	$k_2 = 4.062666$	$k_2 = 5.186748$	$k_2 = 6.793813$
1.00	300.06	300.06	300.05	300.08
1.01	278.82	269.85	257.86	245.52
1.02	259.46	243.23	222.47	202.04
1.03	241.77	219.71	192.66	167.21
1.04	225.60	198.89	167.47	139.16
1.05	210.79	180.42	146.09	116.45
1.10	152.93	114.14	77.72	51.73
1.15	114.12	75.55	44.74	25.98
1.20	87.29	52.06	27.62	14.55
1.30	54.43	27.45	12.58	6.02
1.40	36.41	16.26	6.95	3.32
1.50	25.78	10.59	4.44	2.24
1.60	19.14	7.45	3.18	1.73
1.70	14.77	5.57	2.47	1.46
1.80	11.77	4.38	2.04	1.30
1.90	9.65	3.59	1.77	1.20
2.00	8.09	3.04	1.58	1.13
2.50	4.31	1.82	1.18	1.02
3.00	2.96	1.43	1.07	1.00

Table 6. The ARL_1 values for the proposed chart with i = 3 when $r_0 = 370$

c	$a=2$	$a=5$	$a=10$	$a=20$
	$k_1 = 3.480068$	$k_1 = 4.587742$	$k_1 = 5.79097$	$k_1 = 7.35734$
	$k_2 = 2.982044$	$k_2 = 4.293158$	$k_2 = 5.372559$	$k_2 = 6.89495$
1.00	370.12	370.18	370.94	370.66
1.01	342.67	334.91	323.42	306.42
1.02	317.74	303.64	282.86	254.53
1.03	295.05	275.88	248.13	212.43
1.04	274.37	251.15	218.30	178.12
1.05	255.50	229.10	192.60	150.04
1.10	182.42	148.71	107.22	68.01
1.15	134.14	100.61	63.52	34.26
1.20	101.24	70.58	39.81	19.02
1.30	61.66	38.10	18.13	7.57
1.40	40.45	22.79	9.77	3.98
1.50	28.18	14.81	6.03	2.57
1.60	20.64	10.31	4.15	1.92
1.70	15.74	7.60	3.11	1.57
1.80	12.43	5.87	2.48	1.37
1.90	10.11	4.72	2.08	1.25
2.00	8.42	3.92	1.82	1.17
2.50	4.39	2.15	1.26	1.03

4. ADVANTGES OF PROPOSED CHART

As mentioned earlier the proposed control chart is equal to the Shewhart chart when the two control constants are equal (k_1 = k_2) and i = 1. Tables 7 and 8 have been generated for the ARL_1 comparison of the proposed control chart with the Shewhart chart. The efficiency of the proposed chart can be observed by the decreasing pattern of the ARL_1 values; for instance, the ARL_1 of the proposed chart is 147.04 for a = 20 and c = 1.05 whereas the same shift is detected after 170.71 samples on the average for the existing chart as mentioned in Table 7. The efficiency of the proposed chart is checked for all the possible combinations of the different sittings of r_0 = 200, 300 and 370, a = 1, 2, 5, 10 and 20 and shift levels c = 1, 1.01, 1.02, 1.03, 1.04, 1.05, 1.1, 1.15, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.5 and 3.

4.1 Simulation Study

In this section, we will demonstrate the efficiency of the proposed control chart. For this purpose, we will use the simulated data from the gamma distribution. The data is generated and placed in Table 9. The first 20 observations have been generated for in-control process using gamma distribution with a = 2 and b_0 = 1. Next 30 observations have been generated from a shifted parameter of b_1=1.5. Figure 1 shows the proposed control chart with $r_0 = 370$ for the simulated data, which indicates the out-of-control process at 49th (or 29th subgroup after the actual process shift) subgroup.

The Shewhart chart for this data is shown in Figure 2. From this figure, it can be read that all values are within the control limits which indicates that the process is in control. So, we can say that the proposed control chart performs better to detect a shifted process than the Shewhart chart.

4.2 Industrial Example

In this section, the proposed control chart is applied to monitoring of urinary tract infections (UTIs) at a large hospital. The data represents the duration of male UTIs patient at a hospital. Similar data was used by Santiago and Smith (2013). The data is known to follow the gamma distribution with shape a = 2. The UTIs data is reported in Table 10.

The control limits of the proposed control chart for UTIs data are given in Figure 3. It can be seen from Figure 3 that the process is in-control although some points are close to UCL_2.

Table 7. The Comparison of ARL_1 values for proposed chart with i = 2 and Shewhart Chart when $r_0 = 370$

	$a=1$		$a=2$		$a=5$		$a=10$		$a=20$	
c	Proposed with $k_2=2.83$ $k_2=2.46$	Shewhart $k=2.82$	Proposed with $k_1=3.47$ $k_2=2.96$	Shewhart with $k=3.44$	Proposed with $k_1=4.62$ $k_2=4.07$	Shewhart $k=4.57$	Proposed with $k_1=5.86$ $k_2=5.19$	Shewhart with $k=5.75$	Proposed with $k_1=7.38$ $k_2=6.80$	Shewhart with $k=7.29$
1.00	370.13	370.05	370.02	370.41	370.01	370.06	370.30	370.25	370.17	370.50
1.01	348.68	349.01	343.07	344.82	332.80	335.94	319.30	326.82	304.56	314.37
1.02	328.85	329.54	318.56	321.47	300.00	305.64	276.32	289.40	251.92	268.03
1.03	310.49	311.50	296.23	300.13	271.03	278.68	239.98	257.05	209.46	229.59
1.04	293.47	294.77	275.85	280.60	245.38	254.62	209.14	229.01	175.05	197.55
1.05	277.67	279.23	257.23	262.70	222.62	233.11	182.89	204.61	147.04	170.71
1.10	213.67	216.16	184.89	192.54	140.90	154.23	98.13	121.29	66.08	87.40
1.15	168.11	171.11	136.83	145.21	93.25	106.44	56.67	76.35	33.22	48.93
1.20	134.87	138.11	103.89	112.28	64.20	76.21	34.95	50.61	18.47	29.57
1.30	91.22	94.53	63.95	71.48	33.69	42.82	15.71	25.30	7.38	13.03
1.40	65.16	68.31	42.31	48.74	19.80	26.57	8.47	14.50	3.88	6.99
1.50	48.64	51.54	29.68	35.10	12.75	17.82	5.27	9.25	2.49	4.35
1.60	37.64	40.29	21.84	26.41	8.84	12.72	3.65	6.40	1.85	3.03
1.70	30.00	32.41	16.72	20.60	6.52	9.54	2.76	4.74	1.52	2.30
1.80	24.52	26.72	13.23	16.56	5.05	7.46	2.22	3.69	1.33	1.87
1.90	20.47	22.48	10.77	13.65	4.07	6.03	1.88	3.01	1.21	1.59
2.00	17.40	19.24	8.97	11.48	3.40	5.01	1.66	2.53	1.14	1.41
2.50	9.40	10.65	4.64	6.07	1.92	2.66	1.19	1.51	1.02	1.08
3.00	6.27	7.18	3.12	4.05	1.46	1.87	1.07	1.21	1.00	1.02

Table 8. The Comparison of ARL_1 values for proposed chart with i = 3 and Shewhart Chart when $r_0 = 370$

c	$a=1$		$a=2$		$a=5$		$a=10$		$a=20$	
	Proposed with $k_1=2.84$ $k_2=2.45$	Shewhart with $k=2.82$	Proposed with $k_1=3.48$ $k_2=2.98$	Shewhart with $k=3.44$	Proposed with $k_1=4.58$ $k_2=4.29$	Shewhart with $k=4.57$	Proposed with $k_1=5.79$ $k_2=5.37$	Shewhart with $k=5.75$	Proposed with $k_1=7.35$ $k_2=6.89$	Shewhart with $k=7.29$
1.00	370.00	370.05	370.12	370.41	370.18	370.06	370.94	370.25	370.66	370.50
1.01	348.35	349.01	342.67	344.82	334.91	335.94	323.42	326.82	306.42	314.37
1.02	328.34	329.54	317.74	321.47	303.64	305.64	282.86	289.40	254.53	268.03
1.03	309.82	311.50	295.05	300.13	275.88	278.68	248.13	257.05	212.43	229.59
1.04	292.65	294.77	274.37	280.60	251.15	254.62	218.30	229.01	178.12	197.55
1.05	276.73	279.23	255.50	262.70	229.10	233.11	192.60	204.61	150.04	170.71
1.10	212.29	216.16	182.42	192.54	148.71	154.23	107.22	121.29	68.01	87.40
1.15	166.51	171.11	134.14	145.21	100.61	106.44	63.52	76.35	34.26	48.93
1.20	133.18	138.11	101.24	112.28	70.58	76.21	39.81	50.61	19.02	29.57
1.30	89.56	94.53	61.66	71.48	38.10	42.82	18.13	25.30	7.57	13.03
1.40	63.63	68.31	40.45	48.74	22.79	26.57	9.77	14.50	3.98	6.99
1.50	47.27	51.54	28.18	35.10	14.81	17.82	6.03	9.25	2.57	4.35
1.60	36.43	40.29	20.64	26.41	10.31	12.72	4.15	6.40	1.92	3.03
1.70	28.93	32.41	15.74	20.60	7.60	9.54	3.11	4.74	1.57	2.30
1.80	23.58	26.72	12.43	16.56	5.87	7.46	2.48	3.69	1.37	1.87
1.90	19.63	22.48	10.11	13.65	4.72	6.03	2.08	3.01	1.25	1.59
2.00	16.66	19.24	8.42	11.48	3.92	5.01	1.82	2.53	1.17	1.41
2.50	8.96	10.65	4.39	6.07	2.15	2.66	1.26	1.51	1.03	1.08
3.00	5.97	7.18	2.99	4.05	1.59	1.87	1.10	1.21	1.00	1.02

Table 9. Simulated data

sub-group #	T_i	T^*	sub-group #	T_i	T^*
1	1.809837	1.218652	26	5.399110	1.754314
2	1.237889	1.073727	27	1.761243	1.207646
3	2.039374	1.268135	28	1.621473	1.174816
4	1.077306	1.025132	29	1.778716	1.211627
5	2.252738	1.310902	30	1.135961	1.043409
6	3.140944	1.464491	31	2.161189	1.292898
7	5.128173	1.724464	32	0.992741	0.997574
8	0.651883	0.867075	33	1.788702	1.213890
9	0.266029	0.643146	34	2.516615	1.360209
10	1.808507	1.218354	35	2.205274	1.301630
11	0.469320	0.777123	36	3.354196	1.496912
12	0.547690	0.818173	37	2.424050	1.343323
13	1.468596	1.136669	38	3.245086	1.480501
14	0.916153	0.971231	39	3.191697	1.472337
15	0.312489	0.678597	40	0.820055	0.936011
16	0.605785	0.846135	41	1.511208	1.147558
17	2.490716	1.355527	42	0.751908	0.909330
18	1.408161	1.120859	43	2.193221	1.299254
19	1.760745	1.207532	44	1.379546	1.113214
20	2.356271	1.330684	45	1.011609	1.003855
21	5.000220	1.710001	46	1.469198	1.136825
22	3.078897	1.454784	47	5.851449	1.801999
23	2.470717	1.351889	48	1.592499	1.167777
24	3.092654	1.456947	49	11.11340	2.231596
25	1.467930	1.136497	50	1.558906	1.159507

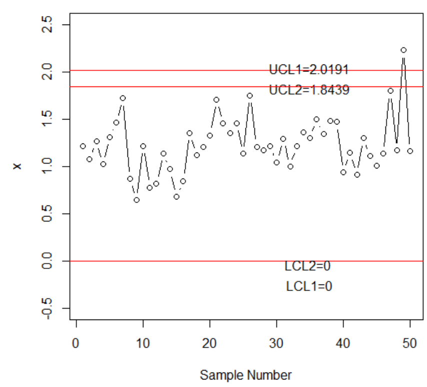

Figure 1. Proposed control chart for the simulated data.

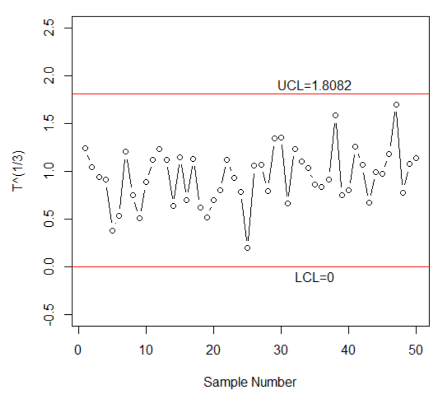

Figure 2. Shewhart control chart for the simulated data.

Table 10. The UTIs data

sub-group #	T_i	T^*	sub-group #	T_i	T^*
1	3.159024	1.467296	26	0.640254	0.861888
2	0.26089	0.638978	27	1.355277	1.106648
3	2.227179	1.305925	28	3.255176	1.482034
4	0.754739	0.91047	29	1.553403	1.158141
5	1.93909	1.246999	30	4.615132	1.664925
6	1.011845	1.003933	31	1.176958	1.055813
7	2.007316	1.261456	32	2.534075	1.363347
8	2.78513	1.40696	33	0.662896	0.87193
9	2.466912	1.351195	34	0.641414	0.862408
10	1.564738	1.160951	35	3.578412	1.529549
11	5.170865	1.729237	36	1.463346	1.135313
12	1.466113	1.136028	37	3.135276	1.46361
13	2.026336	1.265427	38	1.561982	1.160269
14	2.808165	1.410829	39	1.836224	1.224546
15	0.571657	0.829937	40	1.565102	1.161041
16	2.137042	1.288065	41	0.409005	0.742295
17	0.487815	0.7872	42	3.502164	1.518607
18	0.71623	0.894714	43	1.734065	1.201402
19	1.584113	1.165723	44	1.472022	1.137553
20	1.800484	1.21655	45	4.043703	1.593161
21	1.318261	1.096479	46	1.247893	1.076612
22	1.221514	1.068972	47	3.202629	1.474016
23	2.091545	1.278858	48	1.29508	1.090014
24	1.664863	1.185203	49	0.809534	0.931991
25	0.285453	0.658433	50	1.331634	1.100175

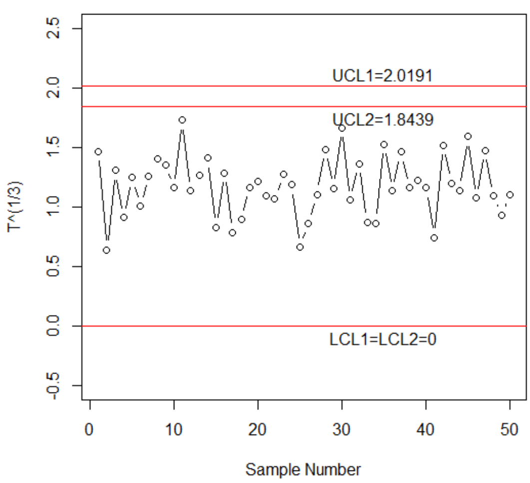

Figure 3. The control chart for UTIs data.

5. CONCLUDING REMARKS

The control chart for the efficient monitoring of the production process has been developed for the multiple dependent state sampling scheme under the gamma distribution. The control chart coefficients have been estimated for various target in-control ARLs. Numerical tables have been constructed for the ARL_0 and ARL_1 values. The proposed chart is found to be comparatively effective for the monitoring of process shifts from the ARL comparison. It has been observed from a simulation study that the proposed scheme is effective for the quick response of the shifted process. A real example is added to explain the application of the proposed chart to a healthcare area. This example shows that the proposed control chart can be also used in health monitoring. The proposed scheme can be extended for other non-normal distributions as a future research.

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