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Evaluation of Modified Non-Normal Process Capability Index and Its Bootstrap Confidence Intervals

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ABSTRACT Process capability index (PCI) is used to quantify the process performance and is becoming an attracted area of research. A variability measure plays an important role in PCI. The interquartile range (IQR) or the median absolute deviation (MAD) is commonly used for a variability measure in estimating PCI when a process follows a non-normal distribution In this paper, the efficacy of the IQR and MAD-based PCIs was evaluated under low, moderate, and high asymmetric behavior of the Weibull distribution using different sample sizes through three different bootstrap confidence intervals. The result reveals that MAD performs better than IQR, because the former produced less bias and mean square error. Also, the percentile bootstrap confidence interval is recommended for use, because it has less average width and high coverage probability.

INDEX TERMS Non-normal distribution, process capability index, interquartile range, median absolute deviation, non-normal, Weibull distribution, bootstrap confidence intervals (BCIs), robust methods.

I. INTRODUCTION

If a process has mean μ and standard deviation σ, then the classical process capability index (PCI), C_p is defined as

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

where USL and LSL represent the upper and lower specification limits, respectively. The implementation of eq. (1) requires that process should follow a normal distribution [1]. However, in case of engineering and reliability related studies, the assumption of normality is often violated. Therefore, the applicability of the classical PCI may not be appropriate [2]–[4]. During the past few decades the focus has been shifted to the usage of modified non-normal PCIs and their associated properties are also well examined [1], [3]–[7]. Among several approaches [1], [4], [6] to deal with non-normality, the quantiles' approach [8], [9] is commonly used in practice [2]. The quantile based estimator of the index, C_p requires the replacement of the standard deviation with two quantiles and is defined as

$$C^*_{Np} = \frac{USL - LSL}{Q\,(0.99865) - Q(0.00135)} \tag{2}$$

where $Q(0.00135)$ and $Q\,(0.99865)$ are the 0.135^{th} and 99.865^{th} quantiles of the corresponding non-normal distribution, respectively. As pointed out by [10] and [11] that the use of quantile based PCIs, for heavily skewed distributions, did not provide accurate results.

The non-normality have significant influence on the efficiency of the classical PCI defined in eq.(1) because the standard deviation is considered meaningful and efficient measure of variability only for a normal distribution [12]. In case of non-normal distributions, there are other measures that performed better than standard deviation because of their robustness properties. Among those robust measures the commonly used are, median absolute deviation (MAD), interquartile range (IQR), and Gini's mean difference (GMD). These robust measures are now used in the construction of control charts using different non-normal distributions and showed better performance than existing methods in the literature [12], [13]. However, the use of these robust measures in PCIs is not very common.

Rodriguez [14] introduced the idea of robust capability indices and used median and MAD as robust estimators of mean and standard deviation, respectively. Later on, [15] highlighted that the efficiency of MAD and quantile- based estimators of index C_p or C_{pk} were poorer than under Beta, standard normal, student-t and Gamma distributions. This may be because the process capability is effected by the tail behavior of each distribution [4], [16]. Therefore, a method that performed well for a particular distribution may give erroneous results for another distribution with different tail behavior [4].

The Weibull distribution is commonly used for industry oriented processes and has a significantly different tail behavior. But a limited study is available in literature where the performance of MAD based estimator of index C_p has been evaluated. Recently, Besseris [17] introduced distribution free PCIs by replacing traditional location and dispersion parameters with median and interquartile range; and concluded that these robust PCIs performed better than classical PCIs in predicting non-confirming items. Thus, the above reported studies help to conclude that the performance of MAD and IQR for a Weibull distribution is still not fully explored. Therefore, in present study, an effort has been made to compare the performance of two robust capability indices based on MAD and IQR under different asymmetric behavior of Weibull distribution. Moreover, the focus has been made to construct bootstrap confidence intervals for these PCIs.

This research work is an extension of the earlier work [1]. In earlier work [1] only one robust method: Gini's mean difference (GMD) was applied for study bootstrap confidence interval for two of capability indices C_p andC_{pk}. Both studies are collectively helpful to make decision for selecting appropriate capability index for improving industrial process.

The remaining paper is organized as follows. In section II, the PCIs based on IQR and MAD method for Weibull distribution are presented. The results of point and interval estimation of modified indices are explained in section III. A real life example is presented in section IV. Some concluding remarks and recommendations for future studies are discussed in the last section.

II. METHODOLOGY

The Weibull distribution is an important distribution to model process capability related studies [1], [4]. Suppose a random variable w showed exponential distribution with mean τ, then a random variable, $x = w^{1/\upsilon}$ would be two parameters Weibull distribution with υ as a shape and τ as a scale parameter. Then its pdf is given as

$$f(x,\upsilon,\tau) = \frac{\upsilon}{\tau}\left(\frac{x}{\tau}\right)^{\upsilon-1} \exp\left(-\left(\frac{x}{\tau}\right)^{\upsilon}\right) \tag{3}$$

A. INTER QUANTILE RANGE (IQR)

The inter- quantile range is defined as

$$IQR = Q_3 - Q_1 \tag{4}$$

The both upper and lower quantiles are found by solving the following integrals:

$$\int_{-\infty}^{Q_3} f(x)\,dx = 0.75 \tag{5}$$

$$\int_{-\infty}^{Q_1} f(x)\,dx = 0.25 \tag{6}$$

The Q_3 and Q_1 for an exponential distribution were $[-\tau \ln(0.25)]$ and $[-\tau \ln(0.75)]$ respectively. Using this, the Q_3 and Q_1 for the Weibull distribution are given as

$$Q_3 = [-\tau \ln(0.25)]^{\frac{1}{\upsilon}} \tag{7}$$

$$Q_1 = [-\tau \ln(0.75)]^{\frac{1}{\upsilon}} \tag{8}$$

Therefore,

$$IQR_{weib} = \tau^{\frac{1}{\upsilon}}\left[\ln(0.25)^{\frac{1}{\upsilon}} - \ln(0.75)^{\frac{1}{\upsilon}}\right] \tag{9}$$

B. PCIs BASED ON IQR AND MAD

The IQR based estimator of index C_p [17] is given by

$$C_{piw} = \frac{USL - LSL}{2 * IQR_{weib}} \tag{10}$$

where the subscript "iw" denotes that the IQR is calculated using Weibull distribution. The formula of C_p using median absolute deviation (MAD) as a measure of variability is defined as [14].

$$C_{pmad} = \frac{USL - LSL}{8.9MAD} \tag{11}$$

where MAD is defined as

$$MAD = b * median\{|x_i - MD|\} \quad i = 1, 2, \ldots\ldots, n \tag{12}$$

The b in (12) is a constant and used for making the parameter of interest as consistent estimator. The term MD in (12) showed the sample median.

C. BOOTSTRAP CONFIDENCE INTERVALS

The commonly used bootstrap confidence intervals (BCIs) are the standard (SB), percentile (PB) and the bias corrected percentile (BCPB) bootstrap method [18]. For these confidence intervals, the bootstrap procedure is explained as follows [19]. Draw a random sample, which consists of n independent and identically distributed random variables $z_1, z_2, z_3, \cdots z_n$, from the distribution of interest $\mathcal{F}$. i.e $z_1, z_2, z_3, \cdots z_n \sim \mathcal{F}$. Let $\hat{\gamma}$ is the estimator of index C_p, which is based on IQR or MAD method. Then,

i. A bootstrap sample, $z_1^*, z_2^*, \cdots z_n^*$, has been drawn from the original sample with mass of $1/n$ at each point.

ii. If Z_m^* is one of the bootstrap samples, where (1 ≤ m ≤ B), then its estimator is given as

$$(\gamma_m)^* = \widehat{\gamma}(z_1^*, z_2^*, \ldots, z_n^*) \quad (13)$$

iii. Each $\widehat{\gamma_m}^*$ will be an estimate of $\hat{\gamma}$. The ascending arrangement of all n^n values of the estimator $\gamma_m{}^*$ will make a complete bootstrapped distribution of $\hat{\gamma}$.

The three BCIs of the required PCIs have been constructed; each based on 1000 bootstrapped resample. These BCIs are described below:

1) SB CONFIDENCE INTERVAL

From $B = 1000$, bootstrap estimates of $\hat{\gamma}^*$, calculate the sample average and standard deviation as

$$\bar{\gamma}^* = (1000)^{-1} \sum_{i-1}^{1000} \hat{\gamma}^*(i) \quad (14)$$

$$S_{\hat{\gamma}^*}^* = \sqrt{\left(\frac{1}{999}\right) \sum_{i=1}^{1000} (\hat{\gamma}^*(i) - \overline{\gamma^*})^2} \quad (15)$$

The SB $(1-\alpha)$ 100% confidence interval is

$$CI_{SB} = \overline{\gamma^*} \pm Z_{1-\frac{\alpha}{2}} S_{\hat{\gamma}^*}^* \quad (16)$$

where $Z_{1-\frac{\alpha}{2}}$ is obtained by using $\left(1-\frac{\alpha}{2}\right)^{th}$ quantile of the standard normal distribution.

2) PB CONFIDENCE INTERVAL

From the ordered collection of $\hat{\gamma}^*(i)$, choose $100\left(\frac{\alpha}{2}\right)\%$ and the $100\left(1-\frac{\alpha}{2}\right)\%$ points as the end points to calculate PB. Then, confidence interval would be

$$CI_{PB} = \left(\hat{\gamma}^*_{B(\frac{\alpha}{2})}, \hat{\gamma}^*_{B(1-\frac{\alpha}{2})}\right) \quad (17)$$

For a 95% confidence interval with $B = 1000$, it is:

$$CI_{PB} = \left(\hat{\gamma}^*_{(25)}, \hat{\gamma}^*_{(975)}\right) \quad (18)$$

3) BCBP CONFIDENCE INTERVAL

The BCPB approach helps to correct the bias. Since the bootstrap distribution is based on a sample drawn from the entire bootstrap distribution, it led to the generation of either upward or downward bias in the estimator. The BCPB interval has been calculated using the following steps:

i. The probability $p_0 = pr\left(\hat{\gamma}^* \leq \hat{\gamma}\right)$ has been computed using the ordered distribution of $\hat{\gamma}^*(i)$.

ii. Computing cumulative and inverse cumulative distribution functions; Ø and $Ø^{-1}$ of standard normal variable Z. i.e.

$$Z_0 = Ø^{-1}(p_0)$$

iii. The lower and upper percentiles of $\hat{\gamma}^*$ are calculated as

$$P_L = Ø\left(2Z_0 + z_{\frac{\alpha}{2}}\right)$$
$$P_U = Ø\left(2Z_0 + z_{1-\frac{\alpha}{2}}\right)$$

The final form of BCPB confidence interval will be

$$CI_{BCPB} = \left(\hat{\gamma}^*_{(P_L B)}, \hat{\gamma}^*_{(P_U B)}\right) \quad (19)$$

The performance of the three confidence intervals; SB, PB and BCPB were compared using coverage probabilities and average widths. The coverage probability and average width of each BCI are calculated as

$$\text{Coverage Probability} = \frac{(Lw \leq \hat{C}_p \leq Up)}{B} \quad (20)$$

$$\text{Average Width} = \frac{\sum_{i=1}^{B} (Up_i - Lw_i)}{B} \quad (21)$$

where Lw and Up are $(1-\alpha)$ % confidence interval based on B=1000 replicates.

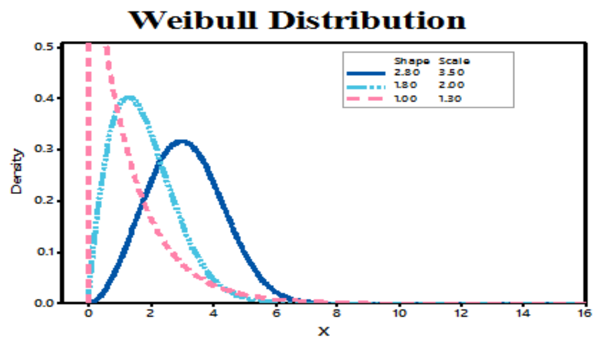

FIGURE 1. **The asymetric behavior of Weibull distribution used for simulation study.**

III. RESULTS AND DISCUSSIONS

The point and interval estimation of IQR and MAD-based estimator of index C_p using simulation are described in this section. For simulation, different sample sizes i.e. $n = 25, 50, 75$ *and* 100 and the combination of shape and scale parameters i.e $[(2.8, 3.5), (1.8, 2.0), (1.00, 1.30)]$ have been used. These combinations of shape and scale parameters represent the low, moderate and high asymmetric behavior of the distribution and are shown in Figure 1. The lower and upper specification limits were taken as [0.0,10.0]. The mean, standard deviation, bias and RMSE for both PCIs under low, moderate and high asymmetric behavior of Weibull distribution are presented in Table 1. The bias is calculated against the widely used standard target value of C_p (equal to 1.33) in the industry which indicates that only 99.73 % of the product is within the 75% of the specification limits [20]. The simulation bias and root mean square error (RMSE) are calculated as

$$Bias = \bar{y} \quad (22)$$

$$RMSE = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2 + (Bias)^2} \quad (23)$$

where y_i represents the value of the estimator with mean $\bar{y}$ and γ is the parameter needed to be estimated. The R-software was used for simulation study.

TABLE 1. The mean, standard deviation, bias and root mean square error of the index Cp using iqr and mad method.

		IQR		MAD	
(Shape, Scale)		Mean(SD)	RMSE(bias)	Mean(SD)	RMSE(bias)
Low (2.8,3.5)	n=25	3.5050 (1.0641)	4.7342 (2.1758)	1.5437 (0.4620)	0.0457 (0.2137)
	n=50	3.2233 (0.6346)	3.5845 (1.8933)	1.4301 (0.2820)	0.0100 (0.1001)
	n=75	3.1317 (0.4839)	3.2462 (1.8017)	1.3996 (0.2218)	0.0048 (0.0696)
	n=100	3.0851 (0.4177)	3.0804 (1.7551)	1.3778 (0.1891)	0.0023 (0.0478)
Moderate (1.8,2.0)	n=25	4.2765 (1.3230)	8.6823 (2.9465)	1.9142 (0.5873)	0.3413 (0.5842)
	n=50	3.9050 (0.8048)	6.6309 (2.5750)	1.7673 (0.3616)	0.1912 (0.4373)
	n=75	3.8004 (0.6297)	6.1029 (2.4704)	1.7246 (0.2821)	0.1557 (0.3946)
	n=100	3.7323 (0.5312)	5.7709 (2.4023)	1.6971 (0.2362)	0.1348 (0.3671)
High (1.0,1.3)	n=25	4.3339 (1.7310)	9.0233 (3.0039)	2.1717 (0.8547)	0.7087 (0.8417)
	n=50	3.9019 (1.0326)	6.6149 (2.5719)	1.9705 (0.5146)	0.4102 (0.6405)
	n=75	3.7589 (0.7978)	5.8998 (2.4289)	1.9006 (0.4028)	0.3256 (0.5706)
	n=100	3.6960 (0.6898)	5.5980 (2.3660)	1.8823 (0.3445)	0.3050 (0.5523)

The results of Table 1 showed that both sample size and asymmetric behavior of the Weibull distribution have significant impact on bias and RMSE for IQR and MAD-based estimator of index C_p. As the sample size increases; both bias and RMSE decreases and mean estimated value of indices close to the target value especially in a case of MAD-method. On the other hand, when asymmetric behavior changes from low to high asymmetry, both bias and RMSE increases especially in the case of IQR-method. The performance of MAD-based estimator of index C_p, under low asymmetric behavior, is very accurate. As the sample size increases, the estimated values getting close to the target values (equal to 1.33) and hence a less bias and smaller mean square error have been observed. However, overestimation happened for target value under moderate and high asymmetry. On the other hand, IQR method produced large bias and RMSE as compared to the MAD method under all asymmetric behavior of the distribution even for large sample size. The comparison of bias and RMSE is presented using radar chart in figure 2 and figure 3 respectively. The results of two methods indicate that IQR-method corresponds to worse estimates under all asymmetric levels. On the other hand, MAD gives better estimates using different conditions. MAD is considered a better measure of variability to deal with non-normality while considering bias and RMSE; when process follows the Weibull distribution.

A. INTERVAL ESTIMATION OF MAD-BASED INDEX C_p

The results of three BCIs are discussed for the only MAD-based estimator of index C_p because it showed less

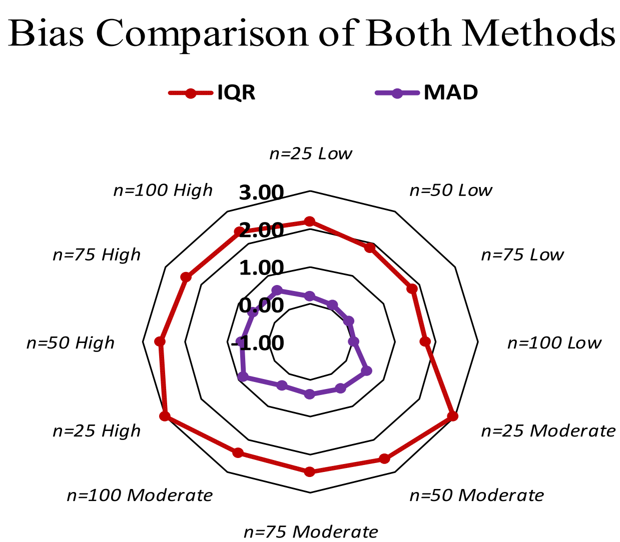

FIGURE 2. Radar chart of bias comparison under low, moderate and high asymmetry.

bias and RMSE. The 95% confidence limits of three methods are presented in Table 2. The coverage probability is reported underneath of each interval. The results revealed that the average width of all confidence intervals reduces when the sample size increases in all cases under study. Moreover, the asymmetric levels affect the average width while the average width increases as asymmetric nature of distribution increases.

From the results of BCIs, the followings conclusions have been drawn.

TABLE 2. The Bootstrap CIs with coverage probabilities under different asymmetric levels using mad method.

n	*SB*	*PB*	*BCPB*
	Low Asymmetry		
25	(0.7050-2.8914)	(1.0249-3.1103)	(1.0531-3.3069)
	0.9990	0.9760	0.6980
50	(0.9315-2.0352)	(1.0391-2.1363)	(1.0391-2.1363)
	0.9990	0.9990	0.7330
75	(0.8672-1.6262)	(0.9157-1.6802)	(1.3590-2.0496)
	0.9990	0.9990	0.7080
100	(1.0678-1.8643)	(1.1401-1.9386)	(1.3038-2.2225)
	0.9990	0.9990	0.7240
	Moderate Asymmetry		
25	(0.7574-2.9950)	(1.0726-3.2716)	(1.1690-3.5938)
	0.9990	0.9810	0.6980
50	(0.9584-2.0994)	(1.0829-2.2358)	(1.0737-2.1980)
	0.9990	0.9990	0.6760
75	(0.8662-1.6423)	(0.9216-1.7134)	(1.2119-1.9866)
	0.9950	0.9820	0.7170
100	(1.1158-1.9631)	(1.1996-2.0121)	(1.3566-2.3825)
	0.9990	0.9990	0.7140
	High Asymmetry		
25	(0.8642-4.3394)	(1.3309-4.6766)	(1.4832-5.7500)
	0.9990	0.9780	0.7270
50	(1.1265-3.0145)	(1.3225-3.1393)	(1.2586-3.0556)
	0.8050	0.9110	0.6070
75	(0.9834-2.2526)	(1.0849-2.3419)	(1.2957-2.6613)
	0.9970	0.9850	0.7160
100	(1.4297-2.8988)	(1.5386-3.0100)	(1.7404-3.3987)
	0.9990	0.9990	0.7350

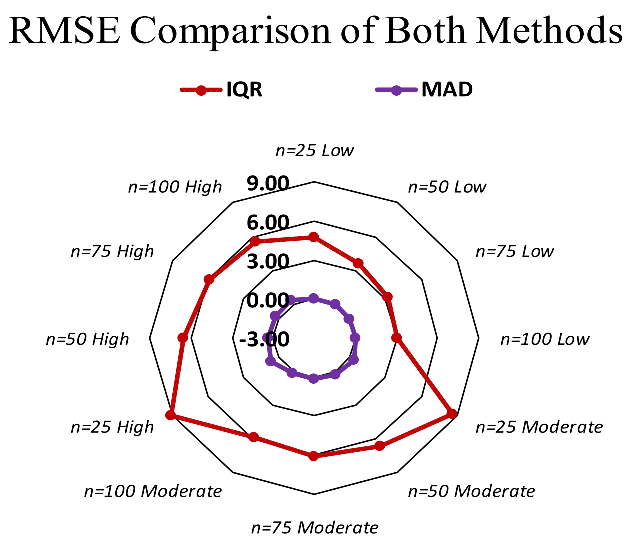

FIGURE 3. Radar chart of RMSE comparison under low, moderate and high asymmetry.

i. The coverage probability is directly proportional to sample size and reached to the nominal level 0.95 for large sample size in the case of *SB* and *PB* method. However, for *BCPB* method it did not reach to a nominal level, particularly for small samples.
ii. Both BCPB and PB CIs showed less average width as compared to SB. Based on the average with; the three bootstrap methods are ranked as $BCPB < PB < SB$.
iii. Among BCPB and PB CIs, former showed lower coverage probability than later. Consequently, PB CI performed better for MAD-method.

TABLE 3. Summary statistics of the data.

Statistic	Value	Statistic	Value
Min.	00.19	AIC_W	140.77
Max.	72.89	AIC_{LN}	140.82
Mean	14.36	AIC_{Ga}	141.23
S.d	18.88	BIC_W	142.67
Q(1)	02.97	BIC_{LN}	142.71
Q(2)	06.50	BIC_{Ga}	143.12
Q(3)	21.90	Shape	0.7707
Skewness	01.66	Scale	12.22
Kurtosis	02.16	USL	75.00
IQR_{Weib}	34.20	LSL	0.00
MAD	05.19	n	19

iv. In all three BCIs, when the transition is made from low to high asymmetric conditions the average width approximately increased by two times. It means under high asymmetry, the width of CI is larger as compared to low and moderate asymmetry.

Based on the low average width and high coverage probability, among three BCIs, the PB CI is recommended under low, moderate and high asymmetric behavior of Weibull process.

B. EXAMPLE

To test the applicability of the MAD and IQR based index, a numerical examples from industry sector is presented in this section. The data is taken from Nelson, W. [21] and is a part of the data set which contains time to breakdown of an insulating fluid between electrodes records

TABLE 4. Bootstrap CIs with their coverage probabilities using MAD methods for Cp.

Method	Cp Value	SB	PB	BCPB
MAD	1.62	(0.5412-4.1855) 0.9990	(0.6067-4.8963) 0.99800	(0.7330-7.3922) 0.7380
IQR	1.09	(0.6745-4.2452) 0.9973	(0.3746-5.2584) 0.9980	(0.2106-3.0469) 0.7580

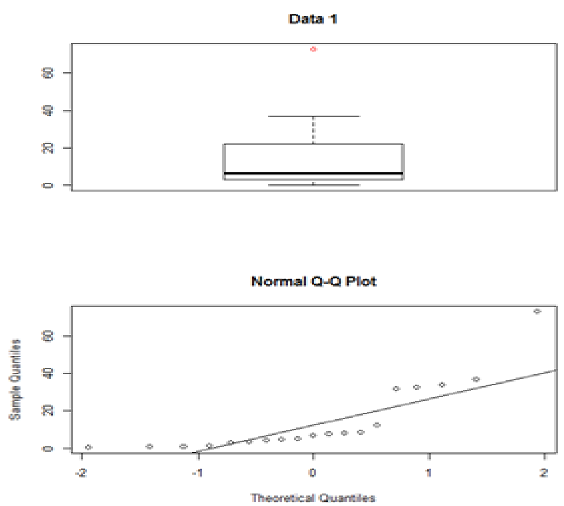

FIGURE 4. Box-Whisker and normal Q-Q plots of data set.

at seven different voltage. Earlier, this data was analyzed by Mukherjee and Singh [22] in order to study the capability of the process. The summary statistics of the data set is reported in Table 3. The Shapiro and Wilks W-test [23] for data is $W = 0.72$ with $p - value = 0.0000$ which indicates that data supports the hypothesis: data follows a non-normal distribution. To confirm that data set follows two parameters the Weibull distribution, the lognormal and gamma distribution is also applied to the data set by using the R-package fitdistrplus [24]. The results are reported in Table 3. The lower value of AIC and BIC showed that Weibull distribution is appropriate as compared to other distributions. The estimates of shape and scale parameters along with upper and lower specification limits are also presented in Table 3.

The box-and-whisker and normal Q-Q plot of the data sets are presented in Figure 4 also supports the above result of W-test. The box-and-whisker plots indicate that data sets have an outlier. Table 4 reports the capability of the two processes using MAD and IQR methods along with three bootstrap confidence intervals. The results of data set under normality showed that the process is not being capable because classical C_p is equal to 0.6620. The normal distribution is not adequate for modeling this data set so the Weibull distribution is used to describe that process. When an adequate model is used; the results of C_p shows that process is being capable. The comparison of proposed C_p values of present study with Mukherjee and Singh [22] ($\hat{I} = 0.977$) showed that both modified indices give batter performance. Furthermore, the MAD based indices are higher than indices based on IQR. The results are not surprising because the data is highly skewed ($S_k \geq 1.5$) and there is also an outlier in the data set. The three BCIs for this dataset are presented in Table 4. The average width and coverage probabilities lead to rank the three BCIs as $PB < SB < BCPB$ in case of MAD-method which also supports the simulation results.

IV. CONCLUSION AND RECOMMENDATIONS

PCIs are important measures for any production process and useful for its continuous improvement. In this study, two robust methods are presented for improving the existing estimation procedures to study the non-normal PCI. The simulation results of point estimation of two methods showed that MAD method is better for the reduction of process variation and yielding high index values. Beside point estimation, interval estimation of MAD-based PCI was constructed because it showed less bias and RMSE. Moreover, three types of bootstrap confidence intervals i.e. *SB, PB*, and *BCPB* and their coverage probabilities using simulation studies were calculated. The selection of the appropriate confidence interval for each method has been made by low average width and higher coverage probability. The simulations illustrated that *PB* CI is recommended for the MAD-based index C_p. Moreover, a real data set example was presented which also support the simulation results. During analysis; the detection of an outlier was also considered while measuring the performance of these indices in the non-normal distributional situation. Numerical outcomes indicate that the proposed indices performed better than existing ones. As a part of future research, the performance of these methods for other advance indices like C_{pk} C_{pm} and C_{pmk} using Weibull and other distributions can be considered.

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