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Event-triggered proportional-derivative control for nonlinear network systems with a novel event-triggering scheme: Differential of triggered state consideration

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Abstract

This article proposes event-triggered proportional-derivative control for a class of nonlinear network control systems. For derivative action of the proposed proportional-derivative control, a novel event-triggering scheme is devised together with the control that considers a differential of a triggered state. The class of the nonlinear network systems is represented as a Lur'e system to consider various nonlinear cases. Time varying transmission delay is considered which can be defined by lower and upper delay bounds. The proposed proportional-derivative control is designed by Lyapunov–Krasovskii stability analysis, and the design condition is presented by linear matrix inequalities. The proposed event-triggered proportional-derivative control and event-triggering condition are verified with numerical simulation.

Keywords

Network control system, nonlinear network system, event-triggered control, event-triggering condition, proportional-derivative control

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Introduction

Traditional point-to-point wiring controls have been transferred to network control system (NCS) areas due to communication development. Studies have accumulated as NCS has developed though some challenges still remain.[1,2] The most widely studied issue is network-induced delay related problems. Network-induced delay reduces control performance and stability of a NCS[3] and deteriorates its transient response.[4,5] Proportional-derivative (PD) controls can be used to improve the transient response, and they can yield higher NCS control performance and stability. Many PD-based controls have been tried in NCS. However, most of them only considers local or remote areas dissociated from network transmission,[6] network-induced delay compensators,[7] or data transmission algorithms.[8,9] Few studies have been done on the PD-based network control itself because of the difficulty in

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implementing the derivative action with network-transmitted data.

Recently PD-based sampled-data synchronization and H_∞ control were proposed by Liu and Lee[10] and Zheng et al.,[11] respectively. Sampled-data control is a methodology for NCS operation.[12] It transmits data only at the sampling instant and holds data during the sampling period. So, stepwise control signals are introduced to the controlled system, and the control input delay increases equivalently as time passes.[13,14] These discontinuity problems are commonly solved by the input-delay approach which regards a sampling holder generating a delayed control input.[15,16] A system is modeled as a continuous system with delayed control input, and control input can be represented as a time-varying delayed signal with nonuniform sampling. The input-delay approach can be used to design a sampled-data controller with Lyapunov–Krasovskii (LK) functional stability criteria, and the resulting condition can be represented by a linear matrix inequality (LMI). The designed sampled-data controller-applied system can be stably controlled when the sampling period is less than the maximum delay for which the LMI is feasible.

Event triggering can conserve communication and computational cost while maintaining control performance of sampling-time triggering.[17,18] Event-triggered control executes control tasks only when a certain event occurs on a system, so data transmission is adapted to the system state. A NCS with network-induced delay can be efficiently controlled using an event-triggering scheme. However, NCSs suffer from not only network-induced delay but also other network-induced imperfections and system intrinsic properties.[19–21] The network-induced imperfections such as packet dropout, signal quantization and the system intrinsic properties such as internal nonlinearity and input and/or output saturation of controllers or sensors are nonlinearity factors of NCSs.[22]

To overcome the network-induced delay, that is, to stabilize a NCS with transmission delay and control input discretization, delay-dependent system stability analyses[23–25] can be used. To overcome the effects from nonlinearities, it is essential to keep a NCS stable when nonlinear factors exist. However, linear approximations are only valid around some operating points, and it is impossible to check the NCS stability for all nonlinear cases. Considering a nonlinear NCS as a Lur'e system can be a good alternative solution that represents a nonlinear system as a linear dynamics which has unknown nonlinearity around it.[22,26,27] The nonlinearity is assumed to have a sector-bound condition, and every system whose nonlinearity exists between two slopes passing through the origin can be expressed as a Lur'e form. Then, the Lur'e representation can be used to replace stability analyses for individual nonlinearity factors which satisfy the given sector condition.[28] Many nonlinear system stability analyses or control studies have borrowed the Lur'e expression.[4,29,30] A Lur'e system can be a good choice for robust nonlinear NCS controllers design.

Although event-triggered controls are enthusiastically being studied, few studies have been done on event-triggering conditions. Periodic event-checking condition[31] and integral-based event-triggering condition[32] were devised to improve network efficiency by enlarging inter-event time. A Lyapunov-based small-gain-approach applied event-triggering condition which allows the condition tuning[33] and a nonfragile controller-design approach applied event-triggering condition[34] were proposed. However, none of them generate a differential of difference between current and triggered states which is essential for the derivative control.

To the best of the authors' knowledge, there has been no PD control study that uses an event-triggering scheme. Event-triggered PD control has not been studied because it is difficult to achieve a feasible solution with triggered state derivative. This article solves this problem by proposing state and state-derivative-considered event-triggering condition, which can adjust triggering dependency between state and state derivative. In addition, the existing sampled-data PD controllers have prospects for improvement. The PD-based sampled-data synchronization by Liu and Lee[10] assumes the sampled state derivative to be zero, and the PD-based sampled-data H_∞ controller by Zheng et al.[11] has a risk of a bilinear matrix inequality problem. The proposed event-triggered PD control fully considers the triggered state derivatives, and the controller design conditions are always organized as LMIs. Thus, the proposed PD control and triggering scheme can be a great solution for both limitations.

This article proposes an event-triggered PD control and an event-triggering condition which consider differential of triggered states. The sequels are organized as follows. The considered problem and used notations are clarified in section *Notation and problem statement*. The proposed triggering condition is fully described in section *State-derivative-considered novel event-triggering scheme*. The proposed event-triggered PD control design process is well organized in section *Main result*. Both the triggering condition and the event-triggered PD control are verified in section *Numerical simulation*, and the whole developed are summarized in section *Conclusion*.

Notation and problem statement

Notation

Standard notations used throughout this article are as follows: $\mathbb{R}^n$, $\mathbb{R}^{m\times n}$, and $\mathbb{Z}^+$ denote a n-dimensional Euclidean space, the set of all $m \times n$ real matrices, and the positive integer set, respectively. The notation $X>Y(X \geq Y)$ means that $X-Y$ is positive definite (positive semidefinite) when both X and Y are symmetric matrices. The superscript T represents transposition of a vector or a matrix. The I denotes an appropriately dimensioned identity matrix. The words *diag* and *sym* are abbreviation for "diagonal matrix" and "symmetrical matrix summation," and subscript "$*$" is used to avoid duplicated similar explanations. Also, matrix dimensions are assumed to be compatible with algebraic operation when they are not explicitly stated.

Problem statement

A class of nonlinear NCSs with time-varying transmission delay τ_k can be defined as a network-connected Lur'e system. The Lur'e system can be defined as follows

$$\begin{cases} \dot{x}(t) &= Ax(t) + Ff(q(t)) + u(t) \\ q(t) &= C_q x(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^n$, and $f(q(t)) \in \mathbb{R}^m$ are state, input, and nonlinearity vectors, respectively, and $A \in \mathbb{R}^{n\times n}$, $F \in \mathbb{R}^{n\times m}$ and $C_q \in \mathbb{R}^{m\times n}$ are known constant matrices. The system-connected network triggers with an event-triggering scheme which will be defined in section *State-derivative-considered novel event-triggering scheme*.

The nonlinearity in equation (1), $f(q(t)) = [f_1^T(q_1(t)), f_2^T(q_2(t)), \ldots, f_m^T(q_m(t))]^T$, is assumed to be memoryless and time invariant. Also, the nonlinearity and its time derivative are assumed to be bounded with the following slope restrictions

$$\alpha_{1,i} \leq \frac{f_i(a)-f_i(b)}{a-b} \leq \beta_{1,i}, \quad a \leq b \tag{2}$$

$$\alpha_{2,i} \leq \frac{\dot{f}_i(a)-\dot{f}_i(b)}{a-b} \leq \beta_{2,i}, \quad a \leq b \tag{3}$$

Here, $1 \leq i \leq m$, and α_* and β_* are bounds of the lower and upper slopes, respectively. The $f(\cdot)$ can be represented as convex combinations of the sector bounds as

$$f_i(q_i(t)) = \left\{\lambda_i^{l_1}(q_i(t))\alpha_{1,i} + \lambda_i^{r_1}(q_i(t))\beta_{1,i}\right\}q_i(t) \tag{4}$$

and

$$\dot{f}_i(q_i(t)) = \left\{\lambda_i^{l_2}(q_i(t))\alpha_{2,i} + \lambda_i^{r_2}(q_i(t))\beta_{2,i}\right\}\dot{q}_i(t) \tag{5}$$

The elements of the $f_i(q_i(t))$ (equation (4)) are defined as

$$\lambda_i^{l_*}(q_i(t)) = \frac{f_i(q_i(t)) - \alpha_{*,i}q_i(t)}{(\beta_{*,i}-\alpha_{*,i})q_i(t)}$$
$$\lambda_i^{r_*}(q_i(t)) = \frac{\beta_{*,i}q_i(t) - f_i(q_i(t))}{(\beta_{*,i}-\alpha_{*,i})q_i(t)}$$

and the elements of the $\dot{f}_i(q_i(t))$ (equation (5)) can be defined in the same manner. Since $\lambda_i^{l_*}(q_i(t)) + \lambda_i^{r_*}(q_i(t)) = 1$, $\lambda_i^{l_*}(q_i(t)) \geq 0$, and $\lambda_i^{r_*}(q_i(t)) \geq 0$, the nonlinearity $f(\cdot)$ can be rewritten as

$$f_i(q_i(t)) = \Lambda_{1,i}(q_i(t))q_i(t) \tag{6}$$

and

$$\dot{f}_i(q_i(t)) = \Lambda_{2,i}(q_i(t))\dot{q}_i(t) \tag{7}$$

where $\Lambda_{1,i}(q_i(t))$ and $\Lambda_{2,i}(q_i(t))$ are elements of convex hulls $Co[\alpha_{1,i}, \beta_{2,i}]$ and $Co[\alpha_{2,i}, \beta_{2,i}]$, respectively. Thus, the nonlinearity, $f(\cdot)$, can be expressed as

$$f(q(t)) = \Lambda_1 q(t) \tag{8}$$

and

$$\dot{f}(q(t)) = \Lambda_2 \dot{q}(t) \tag{9}$$

where

$$\Lambda_* = diag\{\Lambda_{*,1}(q_1(t)), \Lambda_{*,2}(q_2(t)), \ldots, \Lambda_{*,m}(q_m(t))\}$$

Here, the parameters belong to the following combined set of the convex hulls

$$\mathcal{L} \triangleq \{(\Lambda_1, \Lambda_2) | \Lambda_1 \in Co[\Delta_{1,l}, \Delta_{1,u}], \Lambda_2 \in Co[\Delta_{2,l}, \Delta_{2,u}]\} \tag{10}$$

where

$$\Delta_{*,l} = diag\{\alpha_{*,1}, \alpha_{*,2}, \ldots, \alpha_{*,m}\}\text{and}$$
$$\Delta_{*,u} = diag\{\beta_{*,1}, \beta_{*,2}, \ldots, \beta_{*,m}\}$$

We have interest in designing a state-feedback PD controller as

$$u(t) = K_p x(t) + K_d \dot{x}(t) \tag{11}$$

where K_p and K_d are proportional and derivative gains to be determined in section *Main result*. Suppose that the network triggers at $c_k h$ as a certain event occurs on the NCS. Then the triggered signal reaches the system (1) after the transmission time τ_k. Between the triggering instants, the PD-controlled signal at the system side can be defined as

$$u(t) = K_p x(c_k h) + K_d \dot{x}(c_k h)$$
$$\text{for } t \in [c_k h + \tau_k, c_{k+1}h + \tau_{k+1}) \tag{12}$$

where $c_{k+1}h$ is the next triggered sample, and τ_{k+1} is the transmission time of the next triggered state. Then, the closed-loop NCS (equation (1)) with the triggered PD-controlled input (equation (12)) can be redefined in the triggering interval as

$$\dot{x}(t) = Ax(t) + Ff(q(t)) + K_p x(c_k h) + K_d \dot{x}(c_k h),\\ \text{for } t \in [c_k h + \tau_k, c_{k+1}h + \tau_{k+1}) \tag{13}$$

The following assumptions are brought to this article for theoretical development:

Assumption 1. All states of a network-connected control system are measurable.[19,35]

Assumption 2. A sensor is time-triggered with a constant sampling period h, while a controller and a zero-order holder (ZOH) are event-triggered.[18]

Assumption 3. A signal passes through network paths in a single packet, and computational delay of a NCS is negligible.[20]

Assumption 4. The holding interval of the ZOH at the controller is $t \in [c_k h + \tau_k, c_{k+1}h + \tau_{k+1})$, where $k \in \mathbb{Z}^+, c_k h + \tau_k$ is the instant when the control data reaches the ZOH, and τ_k is the total network-induced transmission delay.[18,36] The τ_k is bounded as $0 < \tau_m \leq \tau_k \leq \tau_M$, where τ_m and τ_M are lower and upper bounds of the τ_k, respectively.

The following are important lemmas that will be used in section *Main Result*.

Lemma 1 (Bessel–Legendre integral inequality). For a given matrix $R = R^T > 0$, the following inequality holds for all continuously differentiable functions $x(t)$ in $[a, b] \in \mathbb{R}^n$[23,24]

$$-(b-a)\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \leq -\Gamma_1^T R\Gamma_1 - 3\Gamma_2^T R\Gamma_2 - 5\Gamma_3^T R\Gamma_3 \tag{14}$$

where

$$\Gamma_1 = x(b) - x(a),$$

$$\Gamma_2 = x(b) + x(a) - \frac{2}{b-a}\int_a^b x(s)ds,$$

$$\Gamma_3 = x(b) - x(a) + \frac{6}{b-a}\int_a^b x(s)ds - \frac{12}{(b-a)^2}\int_a^b\int_u^b x(s)dsdu$$

Lemma 2. The following inequality holds for any positive definite symmetric matrix $R = R^T > 0$ and $a \leq s \leq b$[37]

$$\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \geq \frac{1}{b-a}\int_a^b \dot{x}^T(s)dsR\int_a^b \dot{x}(s)ds \tag{15}$$

The proposed event-triggered PD control is designed for a NCS with a time-varying transmission delay $0 < \tau_m \leq \tau(t) \leq \tau_M$. The design condition covers both the lower and upper delay bounds and is summarized as LMIs in *Theorem 1* using the following convexity lemma.

Lemma 3. For proper matrices M_1, M_2, and L, and scalars $\tau_M \geq \tau(t) \geq 0$, the following conditions are equivalent[25]

$$\tau(t)M_1 + (\tau_M - \tau(t))M_2 + L < 0 \tag{16}$$

$$\tau_M M_1 + L < 0, \ \tau_M M_2 L < 0 \tag{17}$$

State-derivative-considered novel event-triggering scheme

This article proposes a novel event-triggering condition for the proposed event-triggered PD controller that uses the derivative of the triggered signal. Figure 1 shows a NCS with the proposed event-triggering condition–triggered PD controller. System states are first measured by a sensor and discretized with a constant sampling cycle h by a sampler. Here, the sampling instants are denoted as s_k, where $s_k \in \Omega = \{s_1, s_2, s_3, \ldots\}$ $k \in \mathbb{Z}^+$ and $\lim_{k\to\infty} s_k h = \infty$. Then, the sampled state of the sensor measurement is obtained as

$$x_s(t) \triangleq x(s_k h), \quad s_k h \leq t < s_{k+1}h \tag{18}$$

An event generator is placed next to the sampler and determines whether to hold or release the sampled signal to network, and resulting signals are fed into the PD controller after passing through the network. The

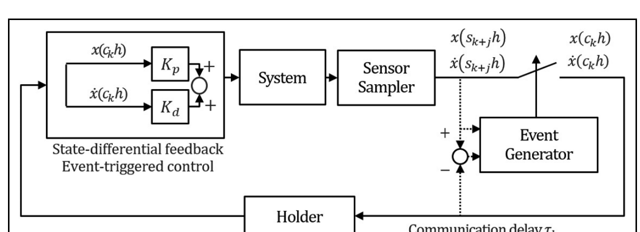

Figure 1. Structure of a NCS with the proposed event-triggering condition–triggered PD controller.

sequence set of the release, that is, event-triggered sample set, is defined as $\Omega_e = \{c_1, c_2, c_3, \ldots\}$ and satisfies $\Omega_e \subset \Omega$. The event-triggered signal is obtained as

$$x_e(t) \triangleq x(c_k h) = x(s_k h) \quad (19)$$

Considering the derivative action of the PD controller, this article proposes the following event-triggering condition

$$\begin{aligned}&\begin{bmatrix} x(s_{k+j}h) - x(c_k h) \\ \dot{x}(s_{k+j}h) - \dot{x}(c_k h)\end{bmatrix}^T \Sigma \begin{bmatrix} x(s_{k+j}h) - x(c_k h) \\ \dot{x}(s_{k+j}h) - \dot{x}(c_k h)\end{bmatrix} \\ &\geq \sigma \begin{bmatrix} x(s_{k+j}h) \\ \gamma \cdot \dot{x}(s_{k+j}h)\end{bmatrix}^T \Sigma \begin{bmatrix} x(s_{k+j}h) \\ \gamma \cdot \dot{x}(s_{k+j}h)\end{bmatrix}\end{aligned} \quad (20)$$

where $j \in \mathbb{Z}^+$; the scalars, $0 \leq \sigma < 1$ and $\gamma > 0$, and the symmetric positive definite weighting matrix, Σ, are design parameters. The time interval between the two consecutive triggered samples can be denoted as $c_{k+1}h - c_k h$ and may consist of multiple sampling cycles h. Under the event-triggering condition (20), the time between the successive triggered samples can be divided into the following subsets

$$\Psi = \Psi_1 \cup \Psi_2 \cup \cdots \cup \Psi_{\bar{d}} \quad (21)$$

where $\bar{d}$ is the upper bound number of acceptable sampling cycles between the successive triggerings and $\Psi \in [c_k h + \tau_k, c_{k+1}h + \tau_{k+1})$ for $i = 1, 2, \ldots, d-1$. For any consecutive triggering interval between $c_k h$ and $c_{k+1}h$, control sequence time is subject to the set Ψ (equation (21)). The error between the latest triggered sample and the current sample is defined as

$$e(c_k h) \triangleq x(c_k h) - x(s_{k+j}h) \quad (22)$$

Then from equation (20), triggered update time, $c_{k+1}h$, is decided by the proposed event-triggering scheme as

$$\begin{aligned}c_{k+1}h = c_k h + min\Bigg\{ jh \Big| &\begin{bmatrix} e(c_k h) \\ \dot{e}(c_k h)\end{bmatrix}^T \Sigma \begin{bmatrix} e(c_k h) \\ \dot{e}(c_k h)\end{bmatrix} \\ \geq \sigma \begin{bmatrix} x(s_{k+j}h) \\ \gamma \cdot \dot{x}(s_{k+j}h)\end{bmatrix}^T &\Sigma \begin{bmatrix} x(s_{k+j}h) \\ \gamma \cdot \dot{x}(s_{k+j}h)\end{bmatrix}\Bigg\}\end{aligned} \quad (23)$$

Remark 1. The event-triggering condition (23) is related to the system state and its derivative in a discrete instance. A sampled state $[x^T(s_{k+j}h)\,\dot{x}^T(s_{k+j}h)]^T$ satisfying equation (23) will be transmitted, and one insufficient for the threshold of equation (23) will be discarded. Triggering dependency between the state and the state derivative can be adjusted by the design parameter γ. When $\sigma = 0$, all sampled sensor measurements are transmitted to the network, and equation (23) becomes the standard sampling-time triggering scheme.

Main result

In this section, an event-triggered PD controller is designed for a nonlinear NCS that triggers with the proposed event-triggering scheme.

For the design, the event-triggered closed-loop system (13) should be denoted with a delayed-system representation. Define the following piecewise affine function

$$\tau(t) \triangleq t - c_k h, \quad t \in \Psi = [c_k h + \tau_k, c_{k+1}h + \tau_{k+1}h) \quad (24)$$

Then, we have $0 < \tau_m \leq \tau_k \leq \tau(t) \leq \bar{\tau} + h = \tau_M$, and the event-triggered PD-controlled system (13) can be rewritten as

$$\begin{aligned}&\dot{x}(t) = Ax(t) + Ff(q(t)) \\ &\quad + K_p[x(t - \tau(t)) + e(c_k h)] + K_d[\dot{x}(t - \tau(t)) + \dot{e}(c_k h)], \\ &t \in \Psi = [c_k h + \tau_k, c_{k+1}h + \tau_{k+1}h)\end{aligned} \quad (25)$$

The proposed event-triggered PD control design condition is summarized as LMIs in the following theorem. An augmented LK functional is constructed by considering time-varying transmission delay and the delay bounds. The Bessel–Legendre integral inequality (*Lemma 1*) is used to decompose a double integral of the state-derivative term, and *Lemma 3* is used to achieve a convex solution for the lower and upper bounds of the time-varying transmission delay.

Remark 2. In the NCS (Figure 1), system measurements are sampled and selectively transmitted through the network. The digitized samples should be reconstructed as a signal, and the ZOH holds the triggered samples and makes a stepwise control input signal.[2] Control input changes as the triggered sample at $c_k h + \tau_k$, and maintains its value until the next triggered sample reaches to the ZOH, $c_{k+1}h + \tau_{k+1}$. Thus, the control input delay, $\tau(t)$, increases equivalently as time passes at $[c_k h + \tau_k, c_{k+1}h + \tau_{k+1})$.[13,14] Also, it is clear that $\tau(t)$ is piecewise linear and satisfies $\dot{\tau}(t) = 1$ from the definition of $\tau(t)$ (equation (24)).

The LK functional terms $V_4(t)$ and $V_5(t)$ in equation (29) is constructed by taking into account the sawtooth structure characteristic of $\dot{\tau}(t) = 1$.

For simplicity in the subsequent formulation, the following notations are defined:

$e_i \in \mathbb{R}^{9n \times n}(i = 1, 2, \ldots, 9)$ are defined as block entry matrices (e.g. $e_3 = [0 \quad 0 \quad I \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$). The other notations are defined as

$$x_a(t) = \left[x^T(t) \quad f^T(q(t))\right]^T$$
$$\xi(t) = \left[x_a^T(t) \quad x_a^T(t-\tau_M) \quad x_a^T(t-\tau(t)) \quad \dot{x}_a^T(t) \quad \dot{x}_a^T(t-\tau(t)) \quad e^T(c_k h) \quad \dot{e}^T(c_k h) \quad \int_{t-\tau_M}^{t} x_a^T(s)ds \quad \int_{t-\tau_M}^{t}\int_{u}^{t} x_a^T(s)dsdu\right]^T$$

$$\Pi_1 = [e_1 \quad e_8 \quad e_9],$$
$$\Pi_2 = [e_4 \quad e_1 - e_2 \quad \tau_M e_1 - e_8],$$
$$\Pi_3 = [e_3 \quad e_5 \quad e_1 \quad e_8 \quad e_9],$$
$$\Pi_4 = [e_3 \quad e_5], \qquad \Pi_5 = [e_6 \quad e_7]$$

$$\Gamma_1^T = e_1 - e_2, \qquad \Gamma_2^T = e_1 + e_2 - \tfrac{2}{\tau_M}e_8,$$
$$\Gamma_3^T = e_1 - e_2 + \tfrac{6}{\tau_M}e_8 - \tfrac{12}{\tau_M^2}e_9,$$
$$\Gamma_4^T = e_1 + \rho e_4,$$
$$\Gamma_5^T = e_1\bar{A}^T + e_3\bar{K}_{p2}^T - e_4\bar{G}^T + e_5\bar{K}_{d2}^T + e_6\bar{K}_{p1}^T + e_7\bar{K}_{d1}^T$$

$$\bar{A} = \begin{bmatrix} GA & GF \\ GA & GF \end{bmatrix}, \bar{G} = \begin{bmatrix} G & 0 \\ G & 0 \end{bmatrix}, Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^T & Z_{22} \end{bmatrix}$$
$$\bar{K}_{p1} = \begin{bmatrix} \kappa_p \\ \kappa_p \end{bmatrix}, \bar{K}_{p2} = \left[\bar{K}_{p1} \quad 0\right], \quad \kappa_p = GK_p$$
$$\bar{K}_{d1} = \begin{bmatrix} \kappa_d \\ \kappa_d \end{bmatrix}, \bar{K}_{d2} = \left[\bar{K}_{d1} \quad 0\right], \quad \kappa_d = GK_d$$
$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{bmatrix}, \ \bar{\Sigma} = \begin{bmatrix} \Sigma_{11} & 0 & \gamma\Sigma_{12} & 0 \\ 0 & 0 & 0 & 0 \\ \gamma\Sigma_{12}^T & 0 & \gamma^2\Sigma_{22} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\bar{\Lambda}_1 = \begin{bmatrix} 0 & 0 \\ \Lambda_1 C_q & -I \end{bmatrix} \text{ and } \bar{\Lambda}_2 = \begin{bmatrix} 0 & 0 \\ \Lambda_2 C_q & -I \end{bmatrix}$$

Theorem 1. Let the network-induced delay $\tau(t)$ satisfy $0 < \tau_m \leq \tau(t) \leq \tau_M$. For given scalars $\tau_m > 0$ and $\tau_M > 0$ and pre-selected ρ, σ, and γ, a nonlinear NCS (equation (1)) with the proposed event-triggering condition–triggered PD controller (23) (equation (12)) is asymptotically stable, if there exist positive definite symmetric matrices P, Q_1, Q_2, R, Z, Σ, $\bar{\Sigma}$, κ_p, and κ_d, and any real matrices G and N such that following inequalities hold

$$\Phi_1 = \Phi - \frac{1}{\tau_M}(e_1 - e_3)Q_2(e_1 - e_3)^T < 0 \tag{26}$$

$$\Phi_2 = \Phi - \frac{1}{\tau_m}(e_1 - e_3)Q_2(e_1 - e_3)^T + (\tau_M - \tau_m)\cdot \left(e_4Q_2e_4^T + sym(\Pi_3[Z_{12}^T \quad Z_{22}]^T\Pi_2^T)\right) < 0 \tag{27}$$

where

$$\begin{aligned}\Phi = {} & sym(\Pi_1 P \Pi_2^T) - \Pi_3 Z \Pi_3^T + \sigma\Pi_4\bar{\Sigma}\Pi_4^T - \Pi_5\Sigma\Pi_5 \\ & + e_1(Q_1 + sym(N\bar{\Lambda}_1))e_1^T - e_2Q_1e_2^T \\ & + e_3 \cdot sym(N\bar{\Lambda}_1)e_3^T + e_4(\tau_M^2 R + sym(N\bar{\Lambda}_2))e_4^T \\ & + e_5 \cdot sym(N\bar{\Lambda}_2)e_5^T \\ & - \Gamma_1^T R\Gamma_1 - \Gamma_2^T R\Gamma_2 - \Gamma_3^T R\Gamma_3 + sym(\Gamma_4^T\Gamma_5)\end{aligned} \tag{28}$$

Moreover, the proportional and derivative gains of the event-triggered PD controller (12) are given by $K_p = G^{-1}\kappa_p$ and $K_d = G^{-1}\kappa_d$, respectively.

Proof. Consider the following LK functional candidate

$$V(t) \triangleq \sum_{i=1}^{5} V_i(t) \tag{29}$$

where

$$V_1(t) = \xi^T(t)\Pi_1 P\Pi_1^T\xi(t),$$
$$V_2(t) = \int_{t-\tau_M}^{t} \xi^T(s)e_1Q_1e_1^T\xi(s)ds,$$
$$V_3(t) = \tau_M \int_{t-\tau_M}^{t}\int_{u}^{t} \xi^T(s)e_4Re_4^T\xi(s)dsdu,$$
$$V_4(t) = (\tau_M - \tau(t)) \cdot \xi^T(t)\Pi_3 Z\Pi_3^T\xi(t),$$
$$V_5(t) = (\tau_M - \tau(t)) \cdot \int_{t-\tau(t)}^{t} \xi^T(s)e_4Q_2e_4^T\xi(s)ds$$

Apply *Lemma 2* on the $\dot{V}_5(t)$ calculation. The time derivative of $V(t)$ can be computed as follows

$$\dot{V}_1(t) = 2\xi^T(t)\Pi_1 P\Pi_2^T\xi(t),$$
$$\dot{V}_2(t) = \xi^T(t)\left(e_1Q_1e_1^T - e_2Q_1e_2^T\right)\xi(t),$$
$$\dot{V}_3(t) = \tau_M^2\dot{x}_a^T(t)R\dot{x}_a(t) - \tau_M \int_{t-\tau_M}^{t} \dot{x}_a^T(s)R\dot{x}_a(s)ds,$$
$$\dot{V}_4(t) = \xi^T(t) - \left\{\Pi_3 Z\Pi_3^T + 2(\tau_M - \tau(t)) \cdot \Pi_3[Z_{12}^T Z_{22}]^T\Pi_2^T\right\}\xi(t),$$
$$\dot{V}_5(t) \leq \xi^T(t)\left\{(\tau_M - \tau(t)) \cdot e_4Q_2e_4^T - \tfrac{1}{\tau(t)}(e_1 - e_3)Q_2(e_1 - e_3)^T\right\}\xi(t)$$

The $\dot{V}(t)$ can be arranged as

$$\dot{V}(t) \leq \xi^T(t)\{sym(\Pi_1 P \Pi_2^T) - \Pi_3 Z \Pi_3^T + e_1 Q_1 e_1^T - e_2 Q_1 e_2^T + \tau_M^2 e_4 R e_4^T + (\tau_M - \tau(t)) \cdot \left(e_4 Q_2 e_4^T + sym(\Pi_3 [Z_{12}^T Z_{22}]^T \Pi_2^T)\right) - \frac{1}{\tau(t)}(e_1 - e_3) Q_2 (e_1 - e_3)^T \Big\} \xi(t)' - \tau_M \int_{t-\tau_M}^{t} \dot{x}_a(s) R \dot{x}_a(s) ds \quad (30)$$

Applying *Lemma 1* to equation (30) leads to

$$-\tau_M \int_{t-\tau_M}^{t} \dot{x}^T(s) R \dot{x}(s) \leq -\xi^T(t)\left(\Gamma_1^T R \Gamma_1 + \Gamma_2^T R \Gamma_2 + \Gamma_3^T R \Gamma_3\right)\xi(t) \quad (31)$$

and

$$\dot{V}(t) \leq \xi^T(t)\{sym(\Pi_1 P \Pi_2^T) - \Pi_3 Z \Pi_3^T + e_1 Q_1 e_1^T - e_2 Q_1 e_2^T + \tau_M^2 e_4 R e_4^T - \Gamma_1^T R \Gamma_1 - \Gamma_2^T R \Gamma_2 - \Gamma_3^T R \Gamma_3 + (\tau_M - \tau(t)) \cdot \left(e_4 Q_2 e_4^T + sym(\Pi_3 [Z_{12}^T Z_{22}]^T \Pi_2^T)\right) - \frac{1}{\tau(t)}(e_1 - e_3) Q_2 (e_1 - e_3)^T \Big\} \xi(t) \quad (32)$$

Add system dynamics (equation (33)), nonlinearity bound (equation (34)), and event-triggering condition (equation (35)) to the upper bound of $\dot{V}(t)$ (equation (32))

$$2 \cdot \xi^T(t)(e_1 + \rho e_4) \times \left\{\bar{A} e_1^T + \bar{K}_{p2} e_3^T - \bar{G} e_4^T + \bar{K}_{d2} e_5^T + \bar{K}_{p1} e_6^T + \bar{K}_{d1} e_7^T\right\} \times \xi(t) = 0 \quad (33)$$

$$2 \cdot \xi^T(t) \times \left(e_1 N \bar{\Lambda} e_1^T + e_3 N \bar{\Lambda} e_3^T + e_4 N \bar{\Lambda}_d e_4^T + e_5 N \bar{\Lambda}_d e_5^T\right)\xi(t) = 0 \quad (34)$$

$$\xi^T(t)\left(\sigma \cdot \Pi_4 \bar{\Sigma} \Pi_4^T - \Pi_5 \Sigma \Pi_5^T\right)\xi(t) \leq 0 \quad (35)$$

Then, upper bound of the $\dot{V}(t)$ can be arranged as

$$\dot{V}(t) \leq \xi^T(t)\{sym(\Pi_1 P \Pi_2^T) - \Pi_3 Z \Pi_3^T + \sigma \Pi_4 \bar{\Sigma} \Pi_4^T - \Pi_5 \Sigma \Pi_5^T + e_1(Q_1 + sym(N\bar{\Lambda}_1))e_1^T - e_2 Q_1 e_2^T + e_3 \cdot sym(N\bar{\Lambda}_1) e_3^T + e_4(\tau_M^2 R + sym(N\bar{\Lambda}_2))e_4^T + e_5 \cdot sym(N\bar{\Lambda}_2) e_5^T - \Gamma_1^T R \Gamma_1 - \Gamma_2^T R \Gamma_2 - \Gamma_3^T R \Gamma_3 + sym(\Gamma_4^T \Gamma_5) - \frac{1}{\tau(t)}(e_1 - e_3) Q_2 (e_1 - e_3)^T \Big\} \xi(t) \quad (36)$$

The upper bound of the $\dot{V}(t)$ (equation (36)) is derived with the time-varying network-induced delay, $\tau(t)$. Applying *Lemma 3*, the $\dot{V}(t)$ upper bound (equation (36)) is divided into two LMI conditions for the delay upper bound τ_M (equation (26)) and the delay lower bound τ_m (equation (27)). The designed event-triggered PD controller with the proposed event-triggering condition (23) stably controls a NCS with network-induced delay that satisfies $\tau(t) \in [\tau_m, \tau_M]$ because of convexity.

This completes the proof.

Remark 3. With the pre-selected ρ and γ, the PD controller gains, K_p and K_d, can be obtained by solving the set of LMIs (26 and 27). Likewise, the σ among the event-triggering parameters can be achieved with the pre-selected Σ, $\bar{\Sigma}$ and feedback gains, K_p and K_d. Therefore, the feedback gains and the event-triggering parameters can be co-designed by employing Theorem 1.

Numerical simulation

A numerical simulation was conducted, and the proposed event-triggering scheme and the event-triggered controller design were verified. A rotational and translational actuator (RTAC) benchmark problem was considered.[38,39] The RTAC system can be represented in the predefined Lur'e form (equation (1)) with the following matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, F = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C_q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Applying *Theorem 1* with $\sigma = 0.1$, $\gamma = 0.1$, and $\rho = 0.3$, maximum and minimum transmission delay bounds are obtained as $\tau_M = 0.7$ and $\tau_m = 0.1$, and convex hulls of the nonlinearity, $f(\cdot)$ (equation (8)), and the differentiated nonlinearity, $\dot{f}(\cdot)$ (equation (9)), are obtained as

$$\Lambda \in Co[diag(0.82, 0.82, 0.82), diag(1.021, 1.021, 1.021)]$$

and

$$\Lambda_d \in Co[diag(0.85, 0.85, 0.85), diag(1.148, 1.148, 1.148)]$$

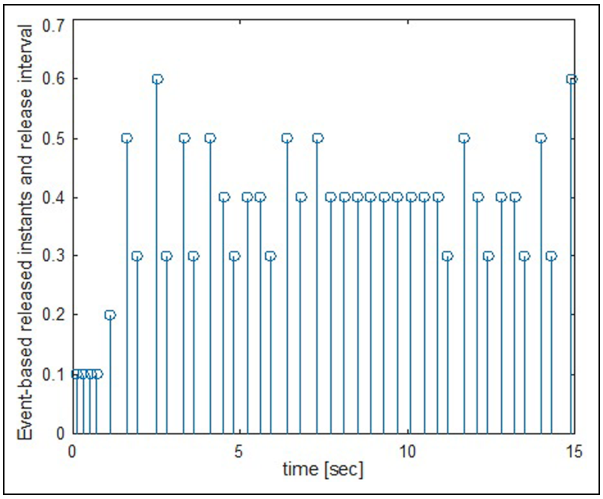

Figure 2. Release instants and release interval of the proposed event-triggering scheme.

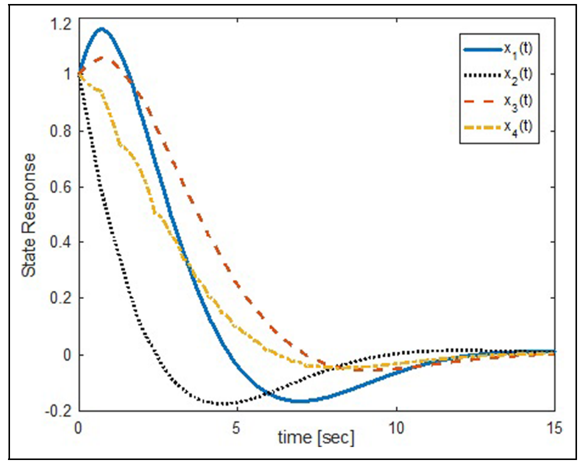

Figure 3. State response of the RTAC using the proposed triggering-scheme-applied event-triggered PD control under $x(0) = [1 \quad 1 \quad 1 \quad 1]^T$.

respectively. The sine function, $f(q(t)) = sin(C_q x(t))$, was used in the simulation. The corresponding proportional and derivative gains are obtained as

$$K_p = \begin{bmatrix} -0.3155 & -0.0469 & 0.0164 & 0.0061 \\ -0.0156 & -0.3236 & 0.0885 & 0.3404 \\ 0.0376 & 0.0366 & -0.3901 & -0.1578 \\ 0.0100 & 0.3821 & -0.2569 & -1.2748 \end{bmatrix},$$

$$K_d = \begin{bmatrix} -0.2208 & -0.0091 & 0.0135 & 0.0054 \\ -0.0560 & -0.3092 & 0.1100 & 0.5911 \\ 0.0820 & 0.0768 & -0.3492 & -0.2123 \\ 0.0676 & 0.7439 & -0.3571 & -1.9588 \end{bmatrix}$$

and design parameter of the event-triggering Σ is obtained as (equation (37))

$$\Sigma = 10^2 \times \begin{bmatrix} 108.2937 & 1.9959 & 0.0851 & 0.0154 & 5.7105 & 0.5919 & 0.1061 & -0.1656 \\ 1.9959 & 107.9484 & -0.8461 & -0.1097 & 2.3898 & 4.0050 & -0.6607 & -2.5321 \\ 0.0851 & -0.8461 & 107.2801 & 3.0042 & -1.0530 & -1.6232 & 6.7025 & 4.2426 \\ 0.0154 & -0.1097 & 3.0042 & 108.3958 & -0.3010 & -3.6852 & 3.5362 & 12.1360 \\ 5.7105 & 2.3898 & -1.0530 & -0.3010 & 104.4162 & 1.0575 & -0.9330 & -0.8550 \\ 0.5919 & 4.0050 & -1.6232 & -3.6852 & 1.0575 & 103.7228 & -1.8157 & -6.8886 \\ 0.1061 & -0.6607 & 6.7025 & 3.5362 & -0.9330 & -1.8157 & 106.3114 & 5.1075 \\ -0.1656 & -2.5321 & 4.2426 & 12.1360 & -0.8550 & -6.8886 & 5.1075 & 119.2375 \end{bmatrix} \quad (37)$$

Release instants and interval (Figure 2) and state response (Figure 3) results are shown. The simulation was conducted with sampling time $h = 0.1$ sec, and initial states of the RTAC were set as $x(0) = [1 \quad 1 \quad 1 \quad 1]^T$. In the simulation run of 15 sec, only 39 sample data packets among the total $15/h = 150$ were triggered.

Conclusion

This article proposes an event-triggered PD control (equation (12)) and a state-derivative-considered event-triggering condition (equation (23)) for a class of nonlinear NCSs (equation (1)) which is represented as a Lur'e system. The proposed event-triggered PD control (equation (23)) considers triggered state derivatives, so it has extended form compared to previous triggering scheme.[17,40] The nonlinearity of the Lur'e system is assumed to be sector bounded (equations (8) and (9)). The proposed PD controller can be designed for all nonlinear NCSs, which satisfies those sector-bound conditions. The proposed event-triggered PD control (equation (12)) and event-triggering scheme (equation (23)) are verified by the numerical simulation.

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